INVOCAB

Mathematics Curriculum Support Manual for Secondary Schools

ACKNOWLEDGEMENTS

"I want to acknowledge my fellow author Ms. Darlene Auguste who was very thorough and methodological in her execution and assistance in this project. I want to thank as well fellow INVOCAB Secondary School teachers: Shelly-Ann Greaves, Nadia Ramlal Kumarsingh, Nigel Maraj and Judith Balkissoon for their timely input towards the end of the exercise. Special thanks go out to my family: Adeltha, Diella, Ervin and Thia Wilson who all contributed in some way, as well as allowed me special time at odd hours to work on the project. Finally, I must thank my deceased parents: Edmund G Wilson (Max) and Theresa A Wilson who were both principals of their own Secondary School and inspired and taught thousands of citizens of Trinidad and Tobago. They, of course, inspired me as well."

- Eugene Garvin Wilson

"I would like to thank God first and foremost for giving me the opportunity to work on this project. I would like to acknowledge my co-author, Mr. Eugene Wilson, whose commitment and expertise made the execution of this project possible. I would like to thank the staff at NIHERST, especially those involved in the INVOCAB project, for their hard work and assistance. I would also like to thank the members of my family and loved ones for their continued support as I undertook this project. A special thank you to my parents, Trevy and Tabita Auguste, for their relentless prayers, support and encouragement in all my endeavours. My mother's thirty plus years of dedication to the teaching service, as well as the diligence of my own teachers, served as inspiration for me to help others succeed on their educational journey."

- Darlene Maria Auguste

"The study of mathematics cultivates the reason; that of the languages, at the same time, the reason and the taste. The former gives grasp and power to the mind; the latter both power and flexibility.

The former, by itself, would prepare us for a state of certainties, which nowhere exists; the latter, for a state of probabilities, which is that of common life.

Each, by itself, does but an imperfect work: in the union of both, is the best discipline for the mind, and the best mental training for the world as it is."

- *Tryon Edwards*[1]

[1] Edwards, MCMLX, p. 397

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AIM OF THE MANUAL

The aim of this manual is to present teachers with non-traditional methods of teaching some of the more challenging topics under the CSEC Mathematics syllabus. The five overarching topics identified include: Measurement, Algebra, Vectors, Functions and Trigonometry.

The strategies presented break free of the traditional rote learning and aim to create a connection between mathematics theory and students' daily experiences. In so doing, students will learn to solve real life problems and cultivate the ability to think logically and critically.

It is hoped that this manual will aid in bringing mathematics alive in secondary school classrooms across both Jamaica and Trinidad and Tobago. The expectation is that students will become active participants in the learning process and begin to develop a deeper understanding and appreciation for Mathematics.

This collection of strategies aims to be a robust toolkit for those teaching Forms 3 to 5. While the manual does focus on objectives at the Form 4 level, it is believed that these strategies can be easily modified to cater to those at the Form 3 and 5 levels respectively.

RATIONALE

"Knowledge of mathematics is an important skill necessary
to succeed in today's world"[2]

Mathematics is a subject rich with information and one that is used more often than realized. However, the teaching of Mathematics is not always welcomed by students. Many students view Mathematics as one of the more challenging subjects. Others may even consider it to be unnecessary since it *seemingly* does not align with a particular 'dream job'. For example, a student who is an aspiring artist may think Mathematics is not required. However, artists require an understanding of angles to create various artistic effects. Similarly, musicians require an understanding of fractions to read and write music. This inability to see the connections between Mathematics and real-world situations may be one of the reasons that some students develop a mental block toward the subject.

A study conducted by Lam (2007) on the teaching of Mathematics in Barbados and Trinidad and Tobago found that while teachers did attempt to make connections to Mathematics and real-world situations, it rarely extended beyond areas of "counting and business"[3]. However, it is important to maximize these real-world associations to help students not only develop an appreciation for and understanding of mathematics, but to also develop essential critical thinking skills. One teacher in the study was able to do this by demonstrating how Mathematics was involved in "patterns on steel pans…, flutes in Hosay festivals [and] wing spans of carnival costumes"[4].

The use of real-world examples was also suggested by Furner, Yahya and Duffy (2005) who present a number of strategies to aid in the teaching of mathematics. Some of the other strategies included using objects as manipulatives, developing

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[2] Furner, Yahya and Duffy, 2005, p. 16
[3] Lam, 2007, p. 75
[4] Lam, 2007, p. 76

lessons around students' experiences and engaging students in verbal discussions to find solutions to mathematics problems[5].

Lam (2007) also highlighted that some students tend to become over reliant on past examination papers. This dependency resulted in students being unable to solve questions when presented in a different format[6].

Students must develop a thorough understanding of underlying concepts so as to be prepared to tackle any question regardless of how it is presented. The lesson plans provided in this manual incorporate real-world examples and other strategies for teaching some of the more challenging topics in the CSEC Mathematics Syllabus. By showing that it is possible to become actively engaged in Mathematics daily, it is hoped that students will realize that Mathematics is certainly not beyond their abilities.

[5] Furner, Yahya and Duffy, 2005
[6] Lam, 2007

Problem Area 1: Measurement

"Measurement is of central importance…because of its power to help children see that mathematics is useful in everyday life and to help them develop many mathematical concepts and skills"[7]

SYLLABUS OBJECTIVES (May-June 2018, pp. 20-21):

<u>Section 4: Measurement</u>

Students should be able to:

- **1.** Convert units of length**,** mass, area, volume, capacity
- **2.** Use the appropriate SI unit of measure for area, volume, capacity, mass, temperature and time (24-hour clock) and other derived quantities
- **5.** Estimate the area of plane shapes
- **8.** Calculate the area of a triangle given two sides and the angle they form
- **12.** Solve problems involving the relations among time, distance and speed
- **13.** Estimate the margin of error for a given measurement
- **14.** Use scales and scale drawings to determine distances and areas
- **15.** Solve problems involving measurement

OBSERVATIONS:

Question 4 of the 2014 CSEC Mathematics Paper 2 focused on the scale of a map. The majority of students attempted the question but "[t]he mean mark was 3.13 out of 10"[8]. According to the Caribbean Examinations Council report, students were able to interpret the scale itself but struggled with its application. The report recommended "authentic examples be developed using maps of the school and immediate community"[9].

[7] National Council of Teachers of Mathematics, 1989, p. 51 cited in Masingila, 1996, p. 4
[8] Caribbean Examinations Council, 2014, p. 4
[9] Caribbean Examinations Council, 2014, p. 5

COMMON ISSUES:

The following are common issues encountered by students when studying Measurement:

- Many students do not use zero on a ruler as the starting point when measuring lengths. This inevitably skews their results and can result in confusion.

- Students have difficulty counting in multiples and thus experience difficulty in drawing and using scales on a graph. Students also tend to exhibit inconsistent use of scales.

- Students sometimes exhibit improper or inaccurate use of protractors.

- Students do not always choose the right tool for the given activity. For example, some students may use a short ruler repeatedly instead of using a long ruler. This could create errors in measurement.

- Students may not have the required instruments for the lesson or may use worn or broken instruments. Worn or broken instruments can lead to incorrect readings for measurement.

- Some students do not have an awareness for impossible measures or answers.

- Students have difficulty with conversion and estimation.

- Some students are unable to see the connection between other aspects of measurement such as area and volume.

- Students sometimes lack a proper or effective revision strategy.

TOPIC OVERVIEW:

What is measurement?

- How many kilometres did John walk today?
- How fast was Danielle walking?

These questions can be answered by measurements. One definition says,

> *"A measurement associates a number with some unit of measure (size or quantity) in order to describe some property of a person or thing."*[10]

Measurements provide information. In order to understand the world around us, both qualitative and quantitative data must be collected. When the scientific method was established, a system of accurate measurement had to be developed in order to collect quantitative data.

An important part of measurement is the unit of measure. A length like centimetre / metre (***metric***) or inch / feet (***imperial***) is used to measure the distance between two points or the distance around something.

All measurements made are approximations. There is a start point, or zero, in measuring and an end point that is represented by a number of some standard (accurately reproducible) unit.

There are many instruments than can be used when determining measurements

Ruler

Tape Measure

Vernier Caliper

Trundle Wheel

The accuracy of a measurement is determined by the ***accuracy*** of the instrument and the ***precision*** in using that instrument.

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[10] Johnson and Glenn, 1964b, p. 3

LESSON 1: MEASURING LENGTHS AND AREAS AND CONVERTING UNITS OF MEASURE

OBJECTIVES: Students should be able to measure and estimate lengths and areas and convert between units of length and those of area with proficiency.

MATERIALS AND RESOURCES:

- Tape measures in metric and imperial units
- Trundle wheel
- Metre rule
- Rulers and pencils
- Projector, laptop and internet (if available)

TEACHING TIME: 2 - 2¼ hours (Three periods or two lessons if necessary)

SET INDUCTION:

- *(If projector and internet are available)* Use Google Earth to zoom into your present location, touching on geography of the Caribbean region and time-zones etc. Identify your school and the building in which the class is situated.
- Make a measurement of a length of a block or field of the school (using the ruler tool on Google Earth). Demonstrate that the units can shift between imperial (traditional units) and metric. Later in the lesson this block/field will be measured directly.
- *(If projector and internet are unavailable)* Ask students to estimate the height of the room in metres and in feet.
- Allow one or two students to demonstrate exactly how this will be done. Give a carpenter's measurement technique if students are unable to do so themselves.
- Ask students to use the tape measure to convert 1 inch to centimetres. For ease of calculation, students may use 1 inch = 2.5 centimetres.

- Look at a picture of downtown Port of Spain from the Lady Young Road (or other relevant landscape) and ask students to estimate which of the prominent buildings is the tallest, second tallest etc. What was the point of reference used?

Figure 1: Port of Spain Landscape

LESSON DEVELOPMENT:

- Indicate to students the importance of the zero in starting to measure.
- Students must also be aware of the need to have a plane/flat surface on which to do linear measurements.
- The smallest fraction of a unit must be identifiable in using the instrument (ruler/tape measure).
- Divide the class into three groups. Each group should have an even number of students, if possible, to facilitate peer work.
- Ask students in Group 1 to measure and record their heights. Ask students to convert these measurements from imperial to metric or vice versa.
- Ask students in Group 2 to measure the length, width and diagonal of a one dollar bill. Ask students to determine if these measurements are different for five or ten dollar bills. Show that measurements will vary slightly between students.

- Ask students in Group 3 to measure a block or field of the school. Where applicable, this should be the same area identified on Google Earth. However, a smaller area may be used if time does not permit. Some students in this group should use a trundle wheel, others a metre rule and others a tape measure. Compare the results.
- Allow each group of students to share their findings and observations. *(Teachers may use the results from Groups 1 and 2 as an opportunity to review mean and mode.)*
- Discuss the terms ***accuracy***, ***precision*** and ***approximation***.
- Ask students if the size of the trundle wheel would have impacted their measurements. Students should identify that a more **accurate** measure could be obtained with a smaller trundle wheel.
- Develop word wall (using the terms in the glossary) as the lesson unfolds.
- Review common prefixes: milli, centi, deci, deca, hecto, kilo
- Connect the prefixes for metre (unit of length)
- Demonstrate the conversion of:
  Kilo ↷ hecto ↷ deca ↷ metre ↷ deci ↷ centi ↷ milli (x10 illustrated)
- Demonstrate the conversion of:
  Milli ↶ centi ↶ deci ↶ metre ↶ deca ↶ hecto ↶ kilo (÷10 illustrated)
- Demonstrate the conversion of
  Kilo^2 ↷ hecto^2 ↷ deca^2 ↷ metre^2 ↷ deci^2 ↷ centi^2 ↷ milli^2 (x100 illustrated)
  Demonstrate the conversion of
- Milli^2 ↶ centi^2 ↶ deci^2 ↶ metre^2 ↶ deca^2 ↶ hecto^2 ↶ kilo^2 (÷100 illustrated)
- Use the concept map at the end of the lesson as a guide to demonstrate how measurement is related to other topics.

GLOSSARY:

Accurate (accuracy):	1) Conforming exactly with the truth or a standard. 2) Refers to the difference/closeness between the measured results of an experiment and the true results of the experiment.
Approximation:	Fairly correct or near to the actual

Error: The degree of inaccuracy in a calculation (e.g. 2% error)

Imperial: (of non-metric weights and measures) still in use in the UK.

Metric system: Decimal measuring system with the metre, litre and gram (or kilogram) as units of length, volume and mass

Perimeter: (1) circumference or outline of a closed figure
(2) outer boundary of an enclosed area

Precision: Refers to how well the results of an experiment or measurement can be duplicated

Prefixes: A prefix is a group of letters placed at the beginning of a word to make a new word with a modified meaning. Applied to SI units it allows other units to be created from the standard ones.

CONVERSION:

Tera – T	x 10^{12}	1 000 000 000 000
Giga – G	x 10^{9}	1 000 000 000
Mega – M	x 10^{6}	1 000 000
Kilo – K	x 10^{3}	1 000
Hecto – h	x 10^{2}	100
Deca – da	x 10^{1}	10
	x 10^{0}	1
Deci – d	x 10^{-1}	0.1
Centi – c	x 10^{-2}	0.01
Milli – m	x 10^{-3}	0.001
Micro – μ	x 10^{-6}	0.000001
Nano – n	x 10^{-9}	0.000000001

EXAMPLES:

Example (1): Convert 5mm to metres.

Answer: 5mm = 5 x 10^{-3}
= 5 x 0.001
= 0.005 m

Example (2): Convert 30 m/s into km/h.

Answer: 30m/s = 30 x 1km/1000 m ÷ (1hr / 3600s)
= 30 x 1/1000 x 3600/1 km/h
= 108 km/h

STUDENT EXERCISE:

(1) Calculate the following:

(a) How many metres are there in 4.78 km?

(b) What is 3,221m in km?

(c) Convert 7.28 L to mL

(d) Convert 3,553 ml to litres (L)

(2) Calculate the following:

(a) Convert 8.27 kg to g

(b) Convert 3,710 g to kg

(c) How many minutes are there in 420 seconds?

(d) How many months are there in 4 years?

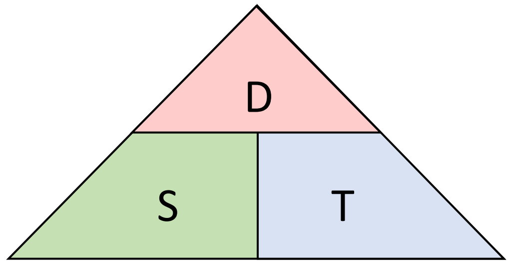
(3) Take 1 mile ≈ 1.61 km and 1 km ≈ 0.621 mile to answer the following:

(a) How many kilometres are equivalent to 3 miles (one lap around the Queens Park Savannah, Trinidad)?

(b) What is 6 km to the nearest mile?

(c) What is 26 miles (a marathon race) to the nearest km?

(d) The speed limit is given in one sign above as well as the distance, in the second sign, from Mt. Hope to San Fernando. If a person travels at the speed limit, how long (time in minutes) will it take to go from Mt. Hope to San Fernando? (Recall: speed = distance ÷ time taken). Why in reality this calculation is unlikely to be true?



(e) If the distance from Port-of Spain to Grand Riviere Bay is 111.8 km. How long will it take to reach the Bay if the average speed for the distance is 60 km/h?

(4) Find the boundaries between which each measurement lies (Hint: plus or minus half of the "nearest" unit).

(a) 80 to the nearest 10

(b) 600 to the nearest 100

(c) 600 to the nearest 1

(d) 14 cm to the nearest 1 cm

(5) Usain Bolt's[11] 100m World Record Times in the 100m and 200m are 9.58s and 19.19s respectively.

(a) What was Bolt's average speed in m/s for his 100m World Record?

(b) What was Bolt's average speed in m/s for his 200m World Record?

(c) What was Bolt's average speed in (a) above converted to km/h?

(d) What was Bolt's average speed in (b) above converted to km/h?

[11] Thrillist Entertainment [Online Image], 2016

(6) A plant grows 11mm every day. How much will it have grown after 12 weeks? Give the answer in cm.

(7) Diella is making pancakes. She is following a recipe[12] that caters for six persons.

Ingredients

- 1 1/2 cups all-purpose flour
- 3 tablespoons sugar
- 1 tablespoon baking powder
- 1/4 teaspoon salt
- 1/8 teaspoon freshly ground nutmeg
- 2 large eggs, at room temperature
- 1 1/4 cups milk, at room temperature
- 1/2 teaspoon pure vanilla extract
- 3 tablespoons unsalted butter, plus more as needed

If she wants to cater for 36 persons instead:

(a) How many cups of flour will she need?

(b) How many cups of milk will be necessary?

(8) The speed gun has been introduced in Trinidad and Tobago to enforce the speed limit which carries a fine of TT$1,000 per ticket. If the accuracy of the measurement is to within 5% of the measured reading.

(a) What is the range of possible measured readings if the actual speed of a vehicle is 80 km/h?

(b) 70 km/h?

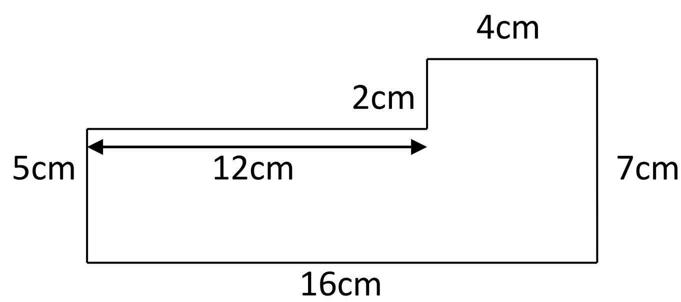
(c) 75 km/h?

The nation's police officers[13] must be aware of the accuracy and precision of the speed guns being used.

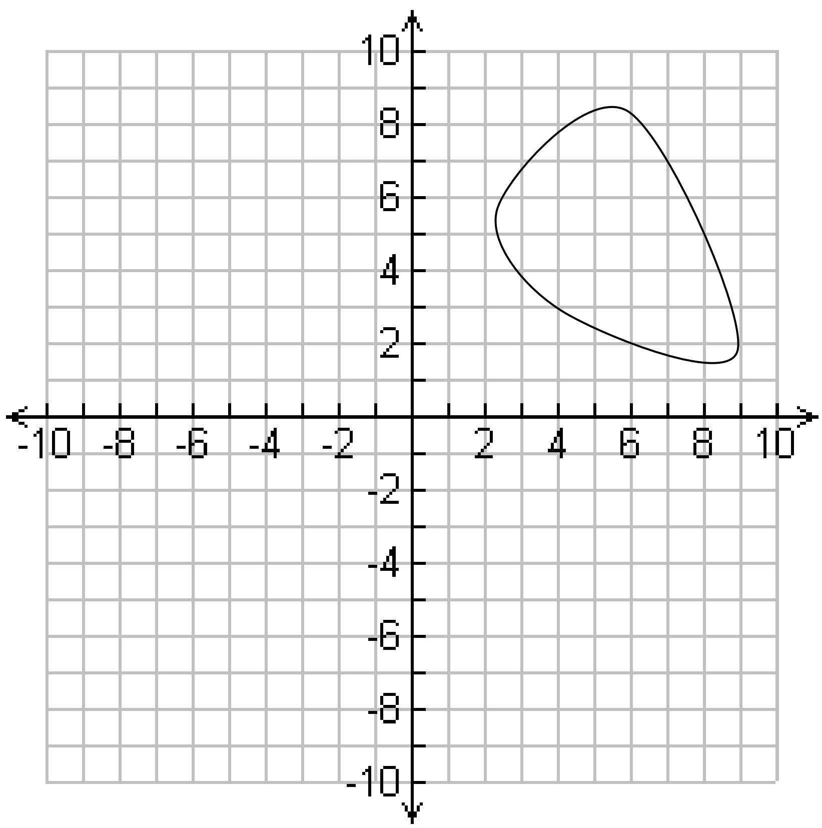
[12] Food Network, 2003

[13] Trinidad and Tobago Express Newspaper [Online Image], 2016

(9) What is the perimeter of the following shape? :



(10) By dividing the shape in question (9) into two rectangles find the area of the shape.



(11) Each of the smallest squares in the figure has an area of 1 cm^2

(a) Estimate the area of the irregular shape shown.

(b) If this diagram represents the outline of a small island in the Caribbean, what is an estimate of its area if each 1cm^2 represents 1 km^2 in reality (scale: 1cm^2 = 1km^2)?

(c) Will the scale 1cm = 1 km be the same for this island representation? When reading a map, the scale may be written as 1 cm : 1 km or perhaps 1: 100,000.

(12) The gate shown above has a deterrent to possible intruders.

(a) Count the number of V shape pieces of ½” round metal pipes that make up half the gate. Double this amount to estimate the total count for both sides of the gate.

(b) If each V shape consists of 2 pieces of 8” of pipe welded together, what is the total length of ½” pipe necessary to do this job?

(c) If each ½” pipe comes in 20 feet lengths (recall 12” = 1 foot: Imperial Unit) how many lengths of steel would be needed?

(d) Go to a nearby hardware and ask to measure the length of any available 20 feet, ½” round pipes. Compare the results.

(13) Perform the following activity:

(a) Measure 10 doors, that look similar, each in cm (to nearest mm) and inches (to nearest 1/16) and compare the range of values.

(b) How do the results for part (a) above affect decisions in door replacement?

(c) Are there standards for these? What does the Bureau of Standards have to say about this?

EXERCISE SOLUTIONS:

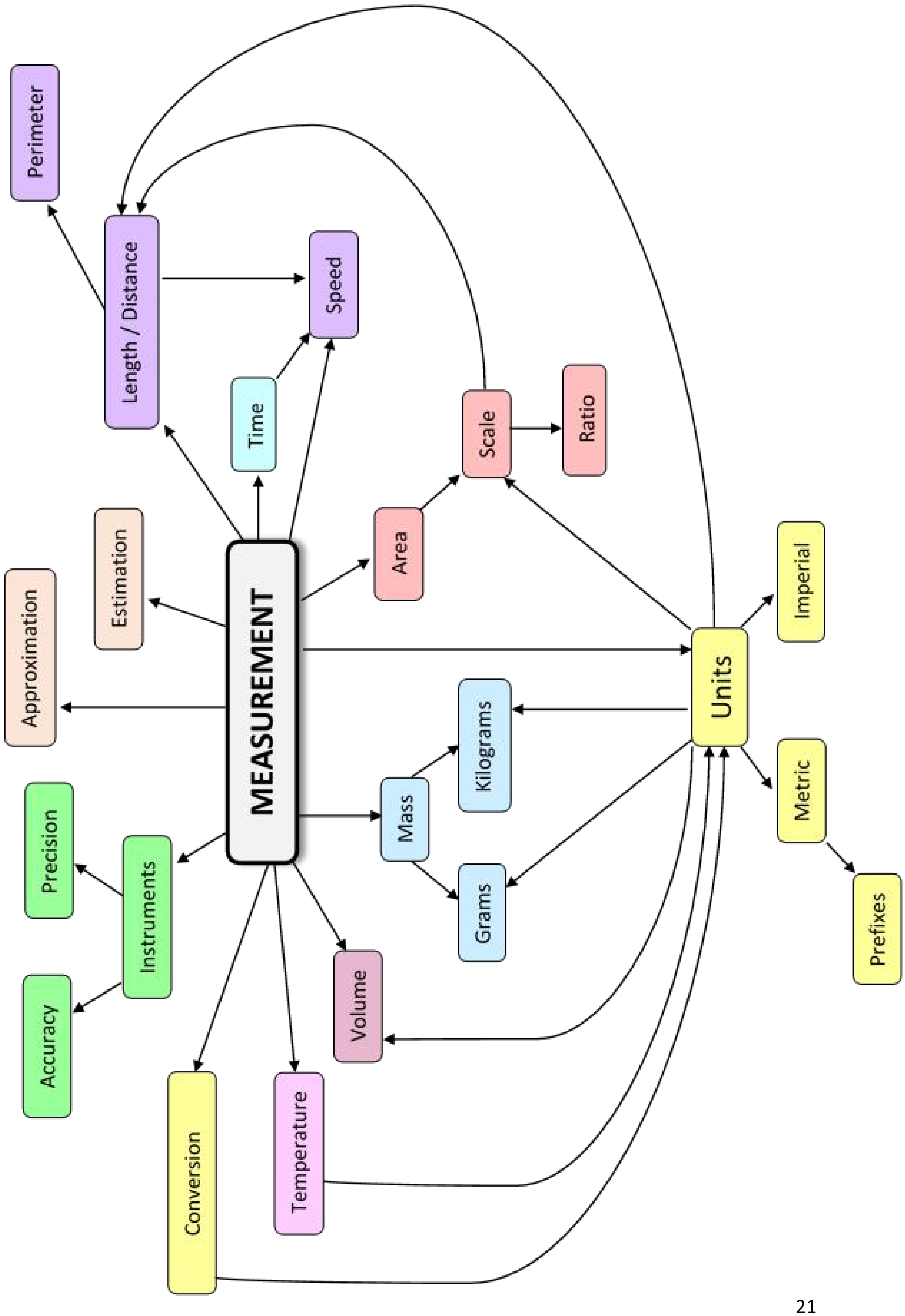
1.	(a) 4,780	(b) 3.221 km	(c) 7,280 ml	(d) 3.553 L	
2.	(a) 8,270 g	(b) 3.710 kg	(c) 7 min.	(d) 48 months	
3.	(a) 4.83 km	(b) 4 miles	(c) 42	(d) 37.5 min.	(e) 1.9 hrs. or 111.8 min.
4.	(a) 75 ≤ X < 85	(b) 575 ≤ X < 625	(c) 599.5 ≤ X < 600.5	(d) 13.9 cm ≤ X < 14.5 cm	
5.	(a) 10.44 m/s	(b) 10.42 m/s	(c) 37.6 km/h	(d) 37.5 km/h	
6.	92.4 cm				
7.	(a) 9 cups flour	(b) 7 ½ cups milk			
8.	(a) 76 to 84 km/h	(b) 66.5 to 73.5 km/h	(c) 71.25 to 78.75 km/h		
9.	46 cm				
10.	88 cm^2				
11.	(a) 29 cm^2	(b) 29 km^2	(c) Yes		
12.	(a) 15 x 2 = 30	(b) 480”	(c) 2 lengths	(d) ≈ 19”	

EVALUATION: Assess students’ understanding of the concepts through evaluation.

TEACHER’S REFLECTION: Reflect on lesson execution and evaluation results. Determine which concepts were not understood or require further explanation.

ADDITIONAL RESOURCES

1) **Video:** How to Measure the Height of Your Walls
https://www.youtube.com/watch?v=0yyjpuWVSAU
2) **Reading:** How to Read a Tape Measure
http://www.johnsonlevel.com/News/TapeMeasure
3) **Video:** How to Use a Tape Measure
https://www.familyhandyman.com/tools/how-to-use-a-tape-measure
4) **Reading:** The Oxford Mathematics Study Dictionary, 2nd Edition
Units and Conversions pp. 150-151


MEASUREMENT
Length / Distance
Perimeter
Speed
Time
Estimation
Approximation
Area
Scale
Ratio
Units
Imperial
Metric
Prefixes
Kilograms
Grams
Mass
Volume
Instruments
Precision
Accuracy
Conversion
Temperature

Problem Area 2: Algebra

"Algebra historically has represented students' first exposure to the abstraction and symbolism that makes mathematics powerful" [14]

SYLLABUS OBJECTIVES (May-June 2018, pp. 26-27):

Section 6: Algebra

Students should be able to:

1. Use symbols to represent numbers, operations, variables and relations
2. Translate between algebraic symbols and worded expressions
3. Evaluate arithmetic operations involving directed numbers
4. Simplify algebraic expressions using the four basic operations
5. Substitute numbers for variables in algebraic expressions
6. Evaluate expressions involving binary operations (other than the four basic operations)

OBSERVATIONS:

Question 2 on both the 2013 and 2014 CSEC Mathematics Paper 02 covered algebraic concepts. In 2013 "less than 1 per cent of [students] earned the maximum available mark" on Question 2 with the average score being "2.33 out of 12"[15]. This improved in 2014 with "6.9 per cent of [students earning] the maximum available mark" and an average of "5.69 out of 12"[16]. Although there was an improvement between the two years, the average score remained below 50 per cent. The transition from concrete to more abstract concepts is often seen as a daunting task and students may develop a mental block to the topic.

[14] Star and Rittle-Johnson, 2009, p. 11
[15] Caribbean Examinations Council, 2013, p. 3
[16] Caribbean Examinations Council, 2014, p. 3

COMMON ISSUES:

The following are common issues encountered by students when studying Algebra:

- Some students have a general fear of letters representing numbers.

- Students often do not realize the level of practice required to fully understand the topic.

- Students encounter difficulty with substituting numbers for letters and simplifying the results. This is even more apparent when squares, cubes, roots and brackets are involved.

- Many students struggle to convert worded questions into algebraic equations or have a general fear of worded questions.

- Some students exhibit an inability to identify slight variations of a known standard expression or equation.

- Students tend to be unfamiliar with the vocabulary of Algebra.

- Students struggle to connect Algebra with other subject areas.

- Students sometimes lack a proper or effective revision strategy.

TOPIC OVERVIEW:

What is algebra?

Algebra is often defined as the "*Language of Mathematics*". It is a language Mathematicians all over the world understand whether English, Spanish, French, Greek, German or any other language is spoken. The challenge lies in making this mathematical language meaningful to the average student. In reality, applications of Algebra can be found in an individual's daily activities.

One definition says, "*Algebra is the language of symbols, operations and relations*"[17]. Another says, "*Algebra is a branch of Mathematics that uses letters etcetera to represent numbers and quantities*"[18].

Algebra also has its own verbs, nouns, adjectives and grammatical sentences. For example:

- Verbs
  - = is read "is equal to"
  - > is read " is greater than" *(reading from left to right)*
  - < is read "is less than" *(reading from left to right)*
- Nouns
  - term
  - expression
  - variable
  - constant
- Adjectives
  - consecutive
  - least
  - most

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[17] The Pocket Oxford Dictionary of Current English, 1992, p. 2

[18] Johnson and Glenn (1964a), p. 2

LESSON 2: SIMPLIFYING EXPRESSIONS AND SUBSTITUTION

OBJECTIVES: Students should be able to simplify algebraic expressions and perform algebraic substitutions

MATERIALS AND RESOURCES:

- Sheets of paper
- Ruler
- Calculator

TEACHING TIME: 1 - 1 ½ hours (Double period)

SET INDUCTION:

- Instruct students to fold a piece of paper in half five times.
- Ask students to reopen the paper and count the number of small rectangles formed. There should be 32 rectangles.
- Ask students to measure the thickness of the paper, that is the thickness of each rectangle (≈ 2mm)
- Ask students to estimate the thickness of the folded paper. Students should discern that the folded paper may be (32 x 2mm ≈ 64mm)
- Ask students to write an expression in "n" for the thickness of the paper after the five folds, if "n" represents the thickness of the paper (i.e each rectangle) in millimetres ($32n$)

LESSON DEVELOPMENT:

- Define and discuss key vocabulary such as, ***terms***, ***like terms***, ***expression***, ***binary operation*** and other vocabulary included in the glossary.
- Perform examples of basic simplification, substitution and evaluation.
- Note common errors among students.
- Compare with Arithmetic to ensure scaffolding and schema development.
- Use the concept map at the end of the lesson as a guide to demonstrate how algebra is related to other topics.

LESSON EXTENSION

- Allow students to discern the relationship between the number of folds and the number of rectangles. In the initial exercise folding the paper in 2, 5 times, resulted in 32 rectangles i.e. $2^5 = 32$. Therefore if *x* represents the number of folds, the number of rectangles = 2^x
- Ask students how thick the folded paper would be if it were possible to fold the paper eight times: in terms of n ($2^8n = 256n$); cm ($256n \div 10 = 25.6n$ cm); m ($256n \div 1{,}000 = 0.256n$ m); km ($256n \div 1{,}000{,}000 = 0.000256n$ km)

GLOSSARY:

Binary Operation:	An operation which combines two objects to produce a third
Constant:	A number that is fixed in value
Equation:	When two expressions are stated as equal to each other an equation is formed
Expression:	Any collection of symbols, figures and signs involving only arithmetical operations
Operation:	A rule (or body of rules) for processing one or more objects. The objects are usually numbers or letters representing numbers.
Operator:	The symbol used to show which operation is being done. Examples: + - x ÷
Terms:	Different parts of an expression connected by plus and minus signs
Variable:	A letter that represents an unknown number

EXAMPLES:

Example (1): Adding like terms

The addition of like terms describes how many of each letter is present.

Therefore $\boldsymbol{a} + \boldsymbol{a} = \mathbf{2}\boldsymbol{a}$

(a) What is $\boldsymbol{a} + \boldsymbol{a} + \boldsymbol{f} + \boldsymbol{f} + \boldsymbol{a} + \boldsymbol{m}$?

Answer: $3\boldsymbol{a} + 2\boldsymbol{f} + \boldsymbol{m}$ (Count the number of each like term)

(b) What is $6\boldsymbol{a} - \boldsymbol{a} - \boldsymbol{a} + \boldsymbol{b} + \boldsymbol{b} - 2\boldsymbol{b} - \mathbf{3}\boldsymbol{b}$?

Answer: $\mathbf{4}\boldsymbol{a} - \mathbf{3}\boldsymbol{b}$

Example (2): Performing operations with brackets

(a) $6(\boldsymbol{x} + 2)$

Answer: $6\boldsymbol{x} + 12$ (Multiply the term outside the brackets by **each** term inside the brackets. Note that the operator between the two terms inside the brackets does not change.)

(b) $\boldsymbol{x}(\boldsymbol{x} + 3)$

Answer: $\boldsymbol{x}^2 + 3\boldsymbol{x}$

(c) $(\boldsymbol{x} - 2)^2$

Answer: This is **not** $\boldsymbol{x}^2 - 2^2$ since an exponent placed outside brackets is **not** applied to each term in the brackets individually. Rather it is applied to the brackets as a whole, that is,

$(\boldsymbol{x} - 2)^2 = (\boldsymbol{x} - 2)(\boldsymbol{x} - 2)$

$= \boldsymbol{x}(\boldsymbol{x} - 2) - 2(\boldsymbol{x} - 2)$

$= \boldsymbol{x}^2 - 2\boldsymbol{x} - 2\boldsymbol{x} + 4$ (Recall: the product of two negatives is positive)

$= \boldsymbol{x}^2 - 4\boldsymbol{x} + 4$

Extension: Ask students to derive the rules $(\boldsymbol{a} + \boldsymbol{b})^2 = \boldsymbol{a}^2 + 2\boldsymbol{ab} + \boldsymbol{b}^2$ and $(\boldsymbol{a} - \boldsymbol{b})^2 = \boldsymbol{a}^2 - 2\boldsymbol{ab} + \boldsymbol{b}^2$

Example (3): Simplification (Multiplication)

(a) $\boldsymbol{b} \times 2\boldsymbol{a}$ Answer: $\boldsymbol{b} \times 2\boldsymbol{a} = 2\boldsymbol{ab}$

(b) $4\boldsymbol{x} \times 3\boldsymbol{y}$ Answer: $4\boldsymbol{x} \times 3\boldsymbol{y} = 4 \times 3 \times \boldsymbol{x} \times \boldsymbol{y} = 12\boldsymbol{xy}$

(c) $5\boldsymbol{a} \times 2\boldsymbol{b}$ Answer: $5\boldsymbol{a} \times 2\boldsymbol{b} = 10\boldsymbol{ab}$

(d) $3\boldsymbol{a} \times 6\boldsymbol{c}$ Answer: $3\boldsymbol{a} \times 6\boldsymbol{c} = 18\boldsymbol{ac}$

Example (4): Simplification (Division)

(a) $6a \div 2$ Answer: $6a \div 2 = 3a$

(b) $6a \div a$ Answer: $6a \div a = 6$

(c) $12x \div 3x$ Answer: $12x \div 3x = 4$

Example (5): Operations

An operation * is defined by $a*b = 3a + b$. Find 2*4 and 5*6

Answer: $2*4 = 3(2) + 4$ $5*6 = 3(5) + 6$

$= 6 + 4$ $= 15 + 6$

$= 10$ $= 21$

Example (6):

Write down the sum of three consecutive numbers if the first number is given by x. Simplify if possible.

Answer: $x + (x + 1) + (x + 2) = 3x + 3$

STUDENT EXERCISE:

- Simplifying terms
- Substitution
- Correct use of directed numbers (pre-requisite)

(1) Simplify the following:

(a) $4x + 5x$ (b) $3a - 2a$ (c) $8x - 4x$ (d) $x - x$

(e) $3ab + 5ab$ (f) $3ab + 6ba$ (g) $ab - ba$ (h) $15xy - 7xy$

(i) $9xy - 3yx$ (j) $7ab - ba$ (k) $5x + 6x + 7x$ (l) $3ab + 4ab + ab$

(m) $a + a + a + a + a$

(2) Fill in the boxes to make the statements true:

(a) $3 \times \square = 12b$ (b) $2a \times \square = 10a^2$ (c) $5p \times \square = 25pq$ (d) $5a \times \square = 10a^2b$

(e) $\square \times 5m^2d = 25m^3d^2$

(3) Simplify the following

(a) $40x \div 5$ (b) $18x \div 3x$ (c) $9m^2 \div 3m$ (d) $25a^2b \div 5a$

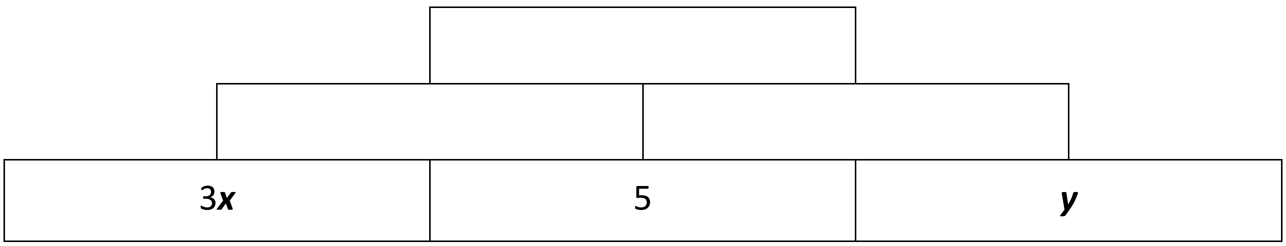
(e) $12a^2b^2 \div 3a^2b$ (f) $18mn^2p^3 \div 2mp^2$

(4) Expand the brackets

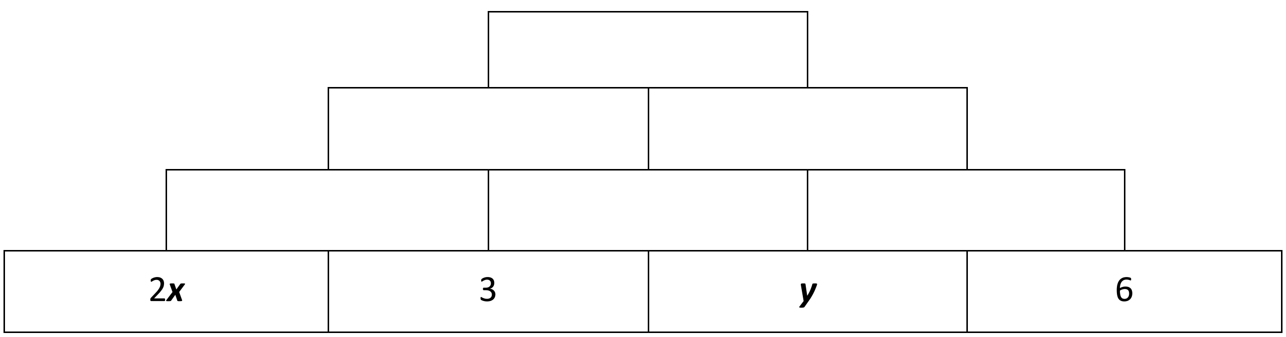
(a) $3(c + 4)$ (b) $4(m – 3)$ (c) $5(4g + 2)$ (d) $3(2m – 5)$

(e) $x(x – 5)$

(5) (a) Fill in the pyramid below by adding adjacent boxes to give the result for the box above



(b) Repeat for the pyramid below



(6) An operation is defined by $m*n = m^2 n^2$. Find:

(a) 3 * 4 (b) 4 * 3 (c) (3*4) * 5 (d) 3 * (4*5)

(7) An operation is defined by $a * b = a^2 + b^2$. Find

(a) 2 * 3 (b) 3 * 2 (c) 3 * 5 (d) (2*3) * 5

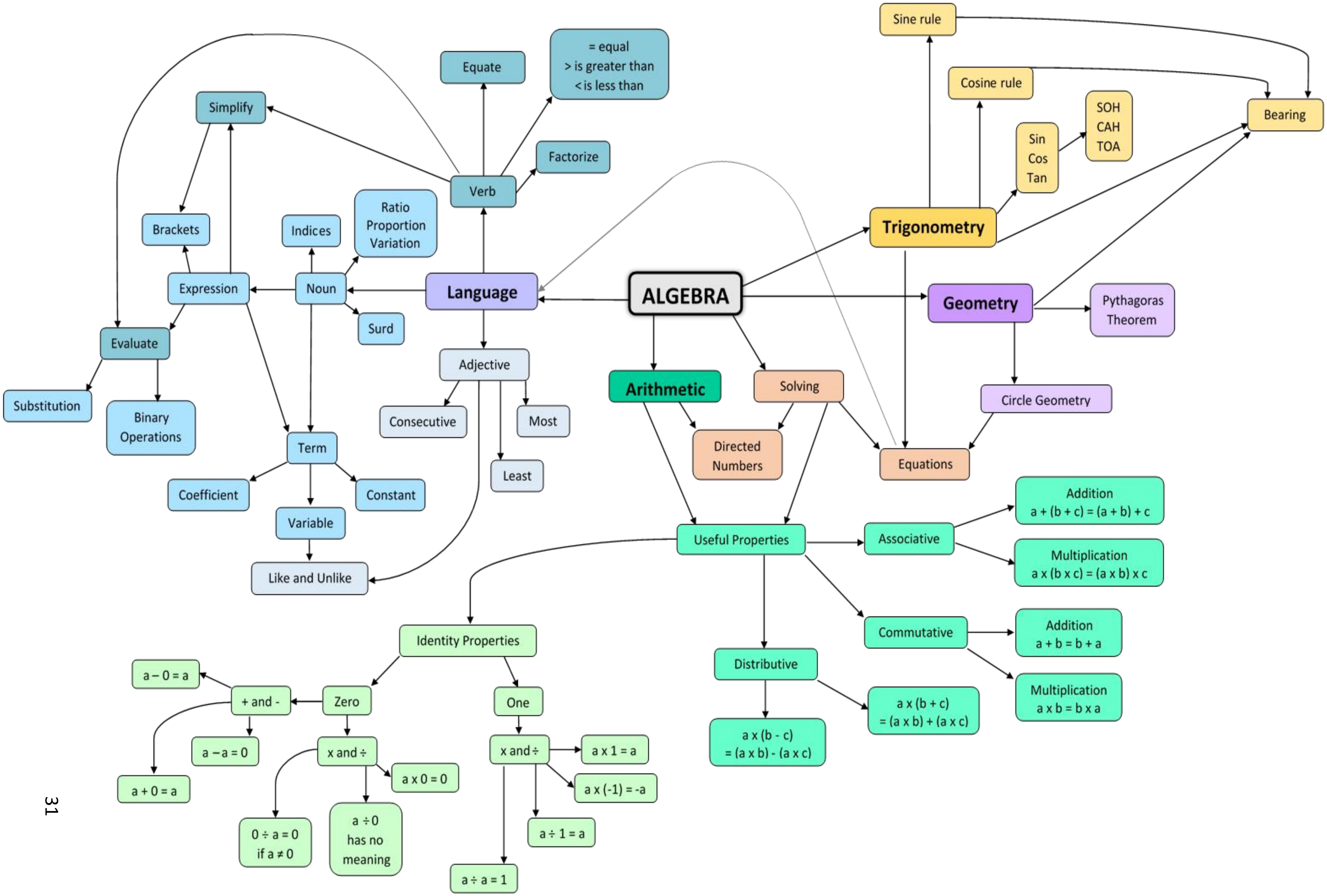
(e) 2 * (3*5)

EXERCISE SOLUTIONS:

1.	(a) 9***x*** (f) 9***ab*** (k) 18***x***	(b) ***a*** (g) 0 (l) 8***ab***	(c) 4***x*** (h) 8***xy*** (m) 5***a***	(d) 0 (i) 6***xy***	(e) 8***ah*** (j) 6***ab***
2.	(a) 4***b***	(b) 5***a***	(c) 5***q***	(d) 2***ab***	(e) 5***md***
3.	(a) 8***x*** (f) $9n^2p$	(b) 6	(c) 3***m***	(d) 5***ab***	(e) 4***b***
4.	(a) 3***c*** + 12	(b) 4***m*** – 12	(c) 20***g*** + 10	(d) 6***m*** – 15	(e) $x^2 - 5x$
5.	(a) *Level 1:* 3***x*** + ***y*** + 10 *Level 2*: 3***x*** + 5; ***y*** + 5 (b) *Level 1:* 2***x*** + 3***y*** + 15 *Level 2*: 2***x*** + ***y*** + 6; 2***y*** + 9 *Level 3:* 2***x*** + 3; ***y*** + 3; ***y*** + 6				
6.	(a) 144	(b) 144	(c) 518,400	(d) 1,440,000	
7.	(a) 13	(b) 13	(c) 34	(d) 194	(e) 1,160

EVALUATION: Assess students' understanding of the concepts through evaluation.

TEACHER'S REFLECTION: Reflect on lesson execution and evaluation results. Determine which concepts were not understood or require further explanation.


ALGEBRA
Language
Arithmetic
Solving
Trigonometry
Geometry
Sine rule
Cosine rule
Sin
Cos
Tan
SOH
CAH
TOA
Bearing
Pythagoras Theorem
Circle Geometry
Equations
Directed Numbers
Useful Properties
Associative
Addition
a + (b + c) = (a + b) + c
Multiplication
a x (b x c) = (a x b) x c
Commutative
Addition
a + b = b + a
Multiplication
a x b = b x a
Distributive
a x (b + c)
= (a x b) + (a x c)
a x (b - c)
= (a x b) - (a x c)
Identity Properties
Zero
One
+ and -
a – 0 = a
a – a = 0
a + 0 = a
x and ÷
a x 0 = 0
a ÷ 0
has no
meaning
0 ÷ a = 0
if a ≠ 0
x and ÷
a x 1 = a
a x (-1) = -a
a ÷ 1 = a
a ÷ a = 1
Verb
Equate
= equal
> is greater than
< is less than
Factorize
Simplify
Brackets
Expression
Evaluate
Substitution
Binary Operations
Noun
Indices
Ratio
Proportion
Variation
Surd
Term
Coefficient
Variable
Constant
Like and Unlike
Adjective
Consecutive
Most
Least

PROBLEM AREA 3: VECTORS

"Vectors are an essential component of the mathematical language of physics"[19]

SYLLABUS OBJECTIVES (May-June 2018, p. 40):

Section 9: Vectors and Matrices

Students should be able to:

1. Explain concepts associated with vectors
2. Simplify expressions involving vectors
3. Write the position vector of a point P (a, b) as $\overrightarrow{OP} = \binom{a}{b}$ where O is the origin (0, 0)
4. Determine the magnitude of a vector

OBSERVATIONS

In 2013 and 2014, Question 11 tested concepts related to vectors. Only 45% of students answered this optional question in 2013 and even fewer, 37%, in 2014. However, in both years, the average mark was particularly low with students' average score being 3.01 and 3.90 out of a total of 15 in 2013 and 2014 respectively[20,21]. Based on these results, it is evident that the topic of vectors is one that is not easily grasped by students. Furthermore, according to the 2018 syllabus, all questions on the Paper 02 examination will be mandatory. It is therefore imperative that students fully understand all topics in the syllabus.

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[19] Knight, 1995, p. 74
[20] Caribbean Examinations Council, 2013
[21] Caribbean Examinations Council, 2014

COMMON ISSUES:

The following are common issues encountered by students when studying Vectors:

- Students sometimes perceive the topic to be difficult because it is assumed that Mathematics is 'supposed' to be difficult.

- The topic of *Vectors* comes across as an abstract topic and is seemingly too far removed from the real world.

- The topic is one that seems more likely to be forgotten by students.

- Students tend to focus solely on what may be tested rather than being aware of real life applications.

- Students struggle to connect the several ways of representing the same vector in different situations.

- Students may develop a 'math phobia' if members of their immediate family also dislike or fear mathematics.

- Students sometimes lack a proper or effective revision strategy.

TOPIC OVERVIEW

What is a vector?

A **vector** is defined as anything that has **<u>both</u>** magnitude (size) and direction. It is graphically depicted by a line with an arrow and its magnitude (size) given by the length of the line. A **scalar**, however, is a quantity that has size only. Examples of scalars and vectors are shown in the table below.

Scalar	Vector
Speed	Velocity
Distance	Displacement
Mass	Weight
	Force
	Upthrust
	Acceleration

Vectors can be found in the external environment.

Figure 2: Guy Wires

The guy wires (support wires) of electric utility poles have forces that balance to keep the pole vertical and stable. These forces can be represented by vectors.

Figure 3: Building Crane

This building crane is made up of cables and poles, among other steel elements that support and balance the forces acting against it. These tensional forces, and in some cases compression forces, keep the crane erect even when it lifts heavy loads. These forces can be modelled using vectors.

Figure 4: Bridge

This bridge over a major highway has a special design to cater for small expansions and contractions. These forces acting on the bridge can also be modelled using vectors.

LESSON 3: PROPERTIES AND BASIC OPERATIONS OF VECTORS

OBJECTIVES: Students should be able to explain basic concepts related to vectors and perform addition and subtraction of vectors.

MATERIALS AND RESOURCES:

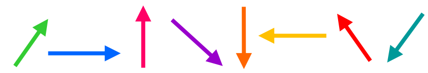
- Grid paper and graph paper
- Tracing paper
- Ruler, pencil and eraser
- Chalk or masking tape
- Projector, laptop and internet (if available)

TEACHING TIME: 1 - 1 ½ hours (Double period)

SET INDUCTION:

- Engage students in a discussion about travelling via airplane. Ask if students have ever travelled during stormy weather or experienced turbulence.
- Explain to students that pilots sometimes have to fly and land planes during adverse wind conditions. These cross-winds or wind-shear have to be carefully navigated to ensure that the planes land safely.
- (*If projector and internet are available)* Use YouTube to present videos on “cross-wind landings” or “wind shear landings” to students.
- Explain that vectors can be used to model the various forces acting on planes during flight. Pilots use a combination of intuition and standard operating procedures to overcome these potentially hazardous situations.

LESSON DEVELOPMENT:

- Define a vector
- Illustrate different ways of representing a vector 
    - A line with an arrow (length of line in proportion to magnitude of the vector and the arrow indicating the direction of the vector).
    - Two capital letters with an arrow head drawn above the letters $\overrightarrow{AB}$
    - Common letters underlined $\underline{a}$, $\underline{c}$, $\underline{n}$ or in bold print when in a textbook or manual **a** , **c** , **n** etc.
    - A column matrix $\begin{pmatrix} x \\ y \end{pmatrix}$ where x represents a horizontal movement along the x-axis and y represents a vertical movement along the y-axis. The two vectors $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 4 \\ 8 \end{pmatrix}$ are illustrated in the Figure 5 below.

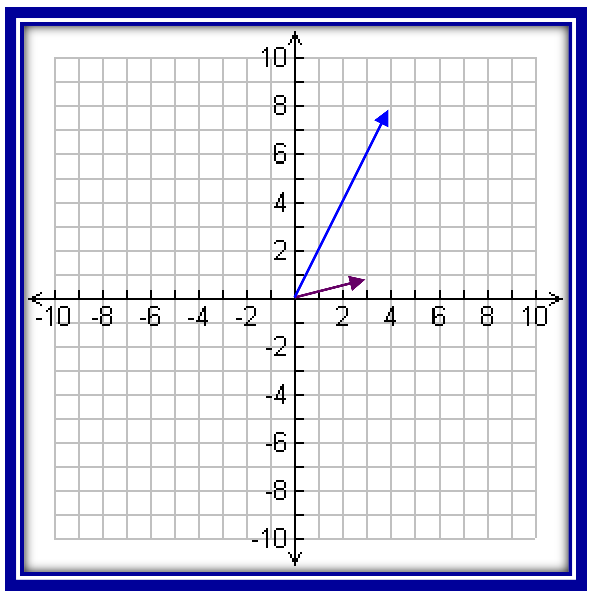


Figure 5

- Demonstrate vector addition
    - Vectors can be added together by putting the head of one vector to the tail of another. This can be done for any number of vectors. The net vector that combines the two is called the ***resultant vector***.
    - $\begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 8 \end{pmatrix} = \begin{pmatrix} 7 \\ 9 \end{pmatrix}$ (as illustrated in Figure 6). The vector addition can also be written: $\underline{a}$ + $\underline{b}$ = $\underline{c}$ as well as $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ (resultant).

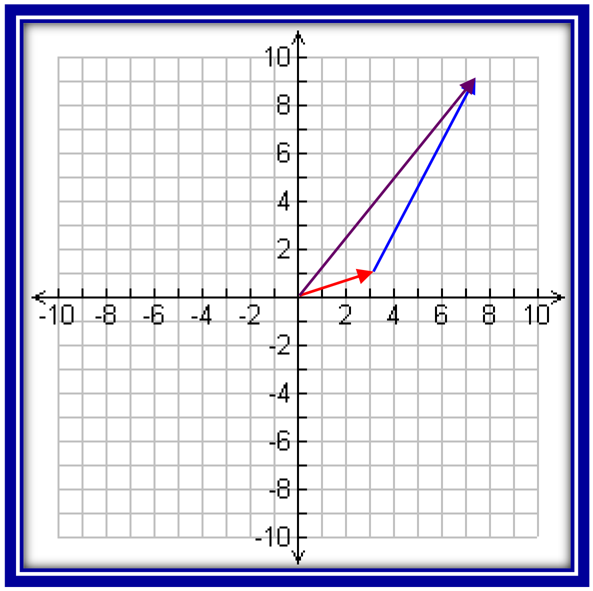


Figure 6

- Demonstrate vector subtraction
  - $\begin{pmatrix}3\\1\end{pmatrix} - \begin{pmatrix}4\\8\end{pmatrix} = \begin{pmatrix}-1\\-7\end{pmatrix}$ which is equivalent to $\begin{pmatrix}3\\1\end{pmatrix} + \begin{pmatrix}-4\\-8\end{pmatrix} = \begin{pmatrix}-1\\-7\end{pmatrix}$.
  - So $\underline{a} - \underline{b} = \underline{a} + (-\underline{b})$. Note that vector $\underline{b}$ has the same size but **opposite direction** to $-\underline{b}$. Call the resultant vector $\underline{d}$, i.e. $\underline{a} - \underline{b} = \underline{d}$.
  - Also $\overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CB}$ (since $\overrightarrow{CB} = -\overrightarrow{BC}$)

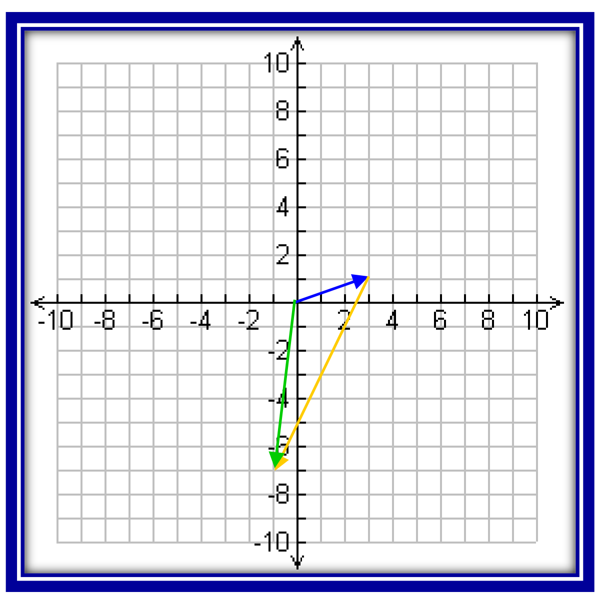
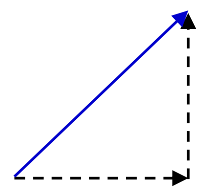


Figure 7

- Determine the length of a vector
    - Since vectors are formed by a horizontal movement, followed by a vertical movement, most vectors (excluding vertical and horizontal vectors) can be seen as the hypotenuse of a right-angled triangle. As such, **Pythagoras' Theorem** can be used to determine the length of a vector.



    - If vector $\overrightarrow{AC} = \begin{pmatrix} 7 \\ 9 \end{pmatrix}$ then by Pythagoras' Theorem its length is $\sqrt{7^2 + 9^2} = 11.4$
    - Therefore, the length of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ is $\sqrt{x^2 + y^2}$
- Use the concept map at the end of the lesson as a guide to demonstrate how vectors are related to other topics.

GLOSSARY[22]:

Column vector (matrix)	A vector in which the set of numbers is written in a vertical line.
Free vector	A free vector is a vector which does not have a definite starting point and so can be placed anywhere in space.
Magnitude	The size or length of a vector.
Null vector / Zero vector	A null vector is a vector which when represented by a line has no length and no direction.
Plane vector	A vector whose direction can be given solely by reference to two-dimensional space.

[22] Tapson, 1999

GLOSSARY[23]**:**

Position vector	A position vector is a vector which starts at some known point, and its finishing point gives a position relative to the starting point, usually (0,0).
Resultant	The vector produced by the addition and/or subtraction of two or more vectors. It is the single vector which can replace all the other vectors and still produce the same result.
Scalar multiplication	The multiplying of the size of a vector by a single number. If the number used is negative, the direction of the vector will be reversed.
Unit vector	A unit vector is a vector which is considered as being the unit of size from which other vectors are made by scalar multiplication.
Vector triangle	A vector triangle is made when 3 vectors are added together to form a triangle whose resultant is zero.

[23] Tapson, 1999

STUDENT EXERCISE:

(1) Depending on the space available, ask students to recreate a grid using chalk in the school yard or masking tape in the classroom. Ask students to walk along the grid to demonstrate the movement along the x and y axes when given a column vector. Once completed, practice drawing column vectors using the 1cm grid paper until students demonstrate competency.

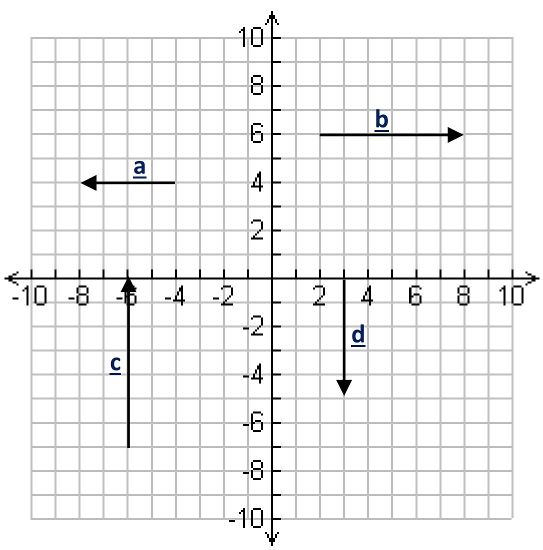
(2) Practice vector additions and subtractions on the 1cm grid.

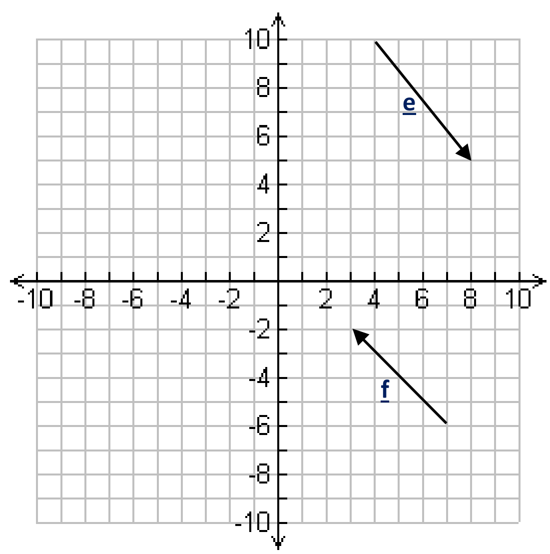
(a) Measure length of resultant vectors with a ruler

(b) Calculate the length of the resultant vectors using Pythagoras' Theorem

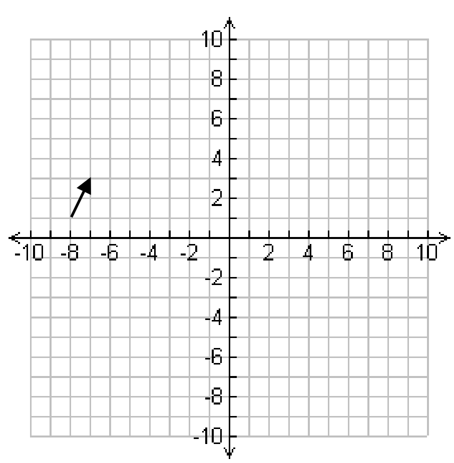
(3) Use tracing paper to trace grid drawings and use alternative vector notations: $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ and $\underline{a} + \underline{b} = \underline{c}$

(4) Write each of the following vectors as a column vector

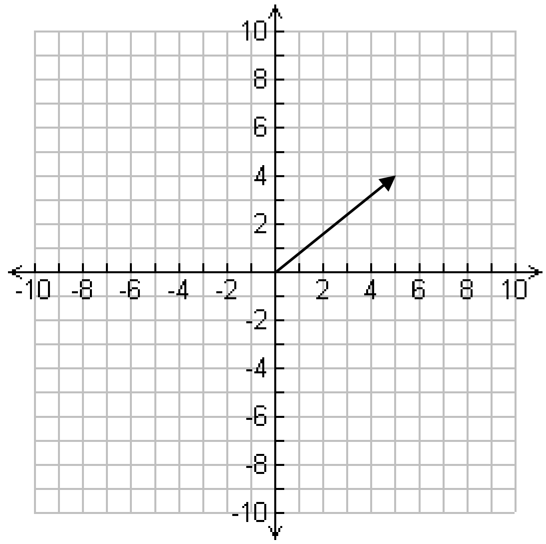




(5) The vector $\underline{a}$ is drawn in the diagram. Draw a vector that is 3$\underline{a}$ and -$\underline{a}$ in the same diagram.

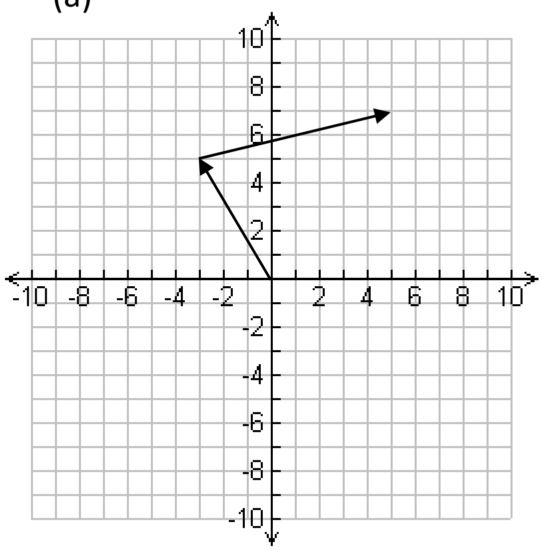


(6) The position vector of P(5,4) is drawn below. Find $\overrightarrow{OP}$ and the <u>length</u> of $\overrightarrow{OP}$ sometimes written $\left|\overrightarrow{OP}\right|$

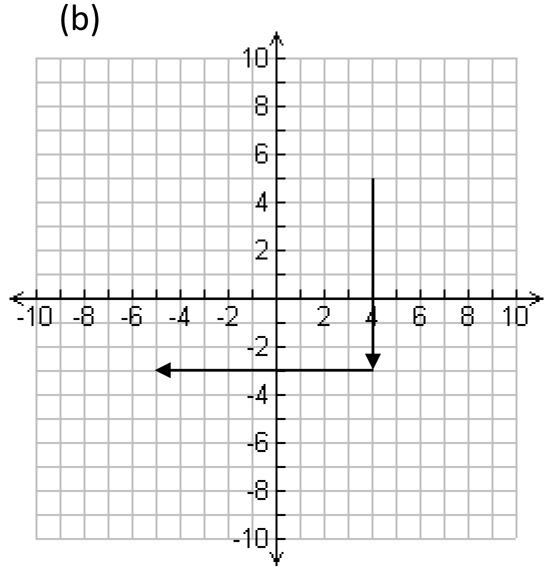


(7) Find the vector sum in each of the two diagrams below. Write each as a column vector first.

(a)

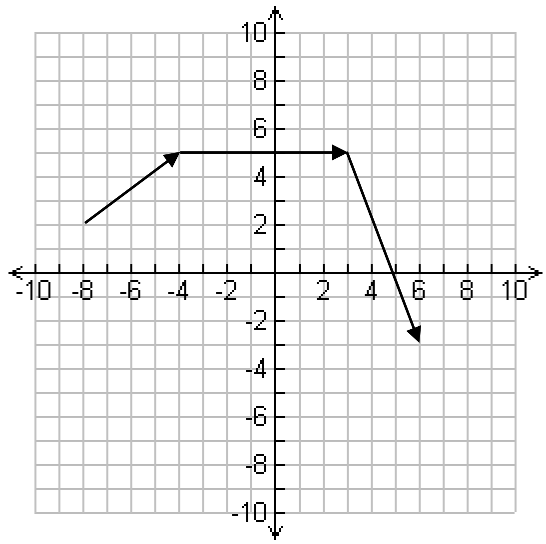


(b)

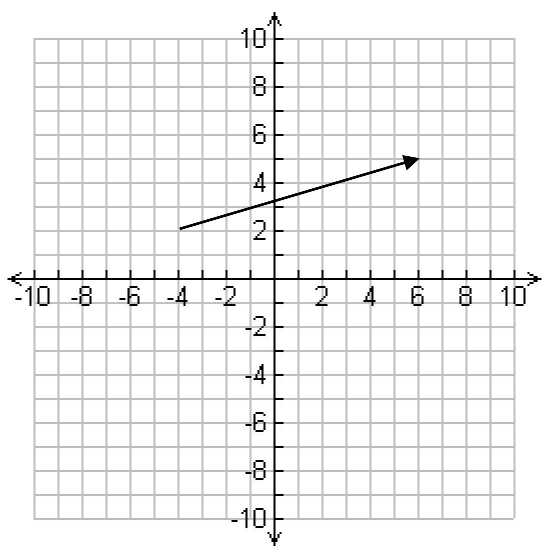


(8) Use tracing paper and trace each pair of vectors in question (7) above together with its resultant. Use the notation $\underline{a} + \underline{b} = \underline{c}$ in the first traced diagram and $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ in the second traced diagram.

(9) Find the sum of the 3 vectors shown below.



(10) Find the length of the vector shown.



EXERCISE SOLUTIONS:

4.	a $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$ f $\begin{pmatrix} -4 \\ 4 \end{pmatrix}$	b $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$	c $\begin{pmatrix} 0 \\ 7 \end{pmatrix}$	d $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$	e $\begin{pmatrix} 4 \\ -5 \end{pmatrix}$
6.	$\lvert\overrightarrow{OP}\rvert$ = 6.40 (3sf)				
7.	(a) $\begin{pmatrix} 5 \\ 7 \end{pmatrix}$	(b) $\begin{pmatrix} -9 \\ -8 \end{pmatrix}$			
9.	$\begin{pmatrix} 14 \\ -5 \end{pmatrix}$				
10.	Vector: $\begin{pmatrix} 10 \\ 4 \end{pmatrix}$ Length: 10.8				

EVALUATION: Assess students' understanding of the concepts through evaluation.

TEACHER'S REFLECTION: Reflect on lesson execution and evaluation results. Determine which concepts were not understood or require further explanation.

ADDITIONAL RESOURCES

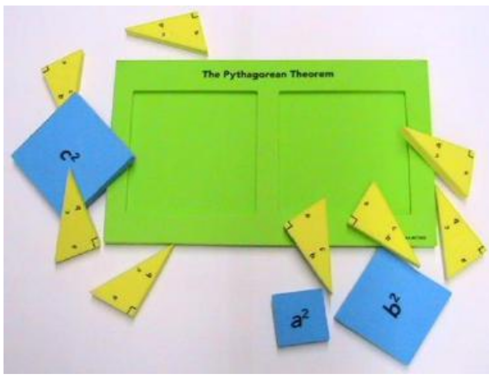
1) **Connect Four**[24] **/ Four-in-a-Line**[25]:

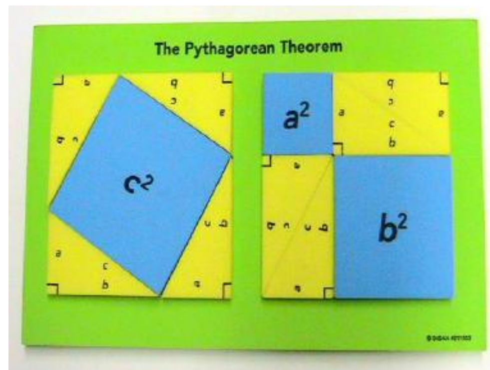
Use this game as a tool to reinforce writing co-ordinates.

[24] Michaels [Online Image], n.d.
[25] MyMaths, 2017

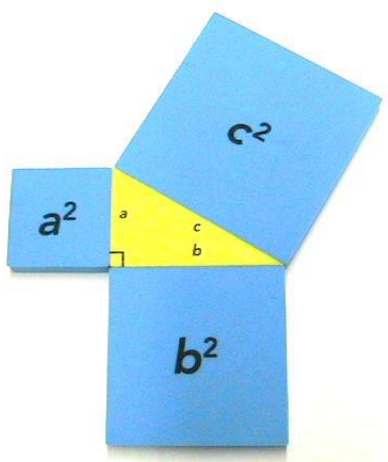
2) **Pythagorean Theorem Tile Set**[26]:

Use this manipulative to help students recall or understand Pythagoras' Theorem. The manipulative consists of two identical square cut-outs that allow the pieces to be placed like a jigsaw puzzle. Solving the puzzle shows four identical yellow triangles in each section of the two square cut-outs. Students will be able to see that the area of c^2 is identical to a^2 plus b^2.

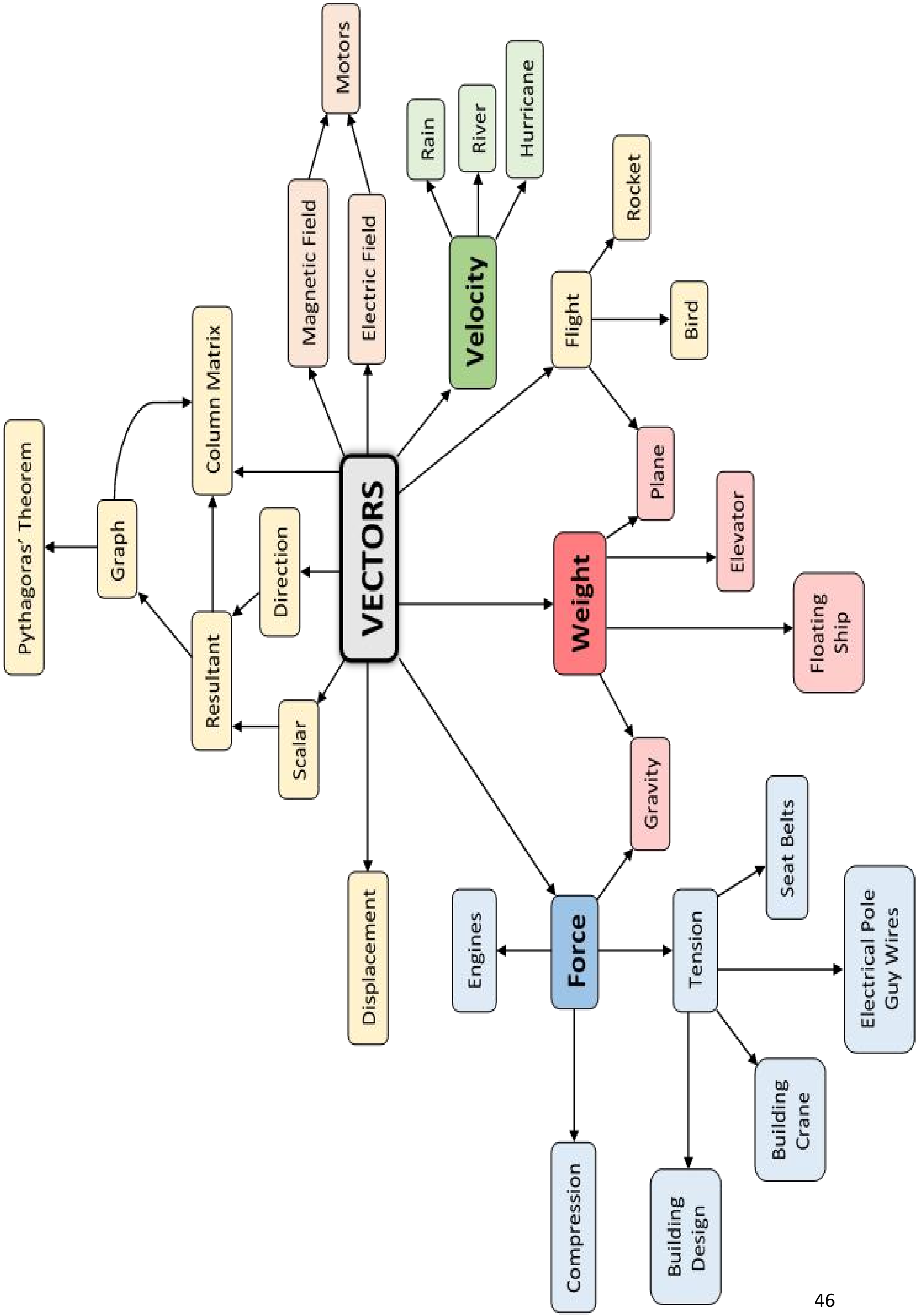




Additionally, by using one yellow right angled triangle, the theorem can be demonstrated using areas with the three blue squares positioned on the sides of the triangle.



[26] Didax Inc., n.d.


VECTORS
Column Matrix
Pythagoras' Theorem
Graph
Direction
Resultant
Scalar
Displacement
Magnetic Field
Electric Field
Motors
Velocity
Rain
River
Hurricane
Flight
Rocket
Bird
Plane
Weight
Elevator
Floating Ship
Gravity
Force
Engines
Tension
Seat Belts
Electrical Pole Guy Wires
Building Crane
Building Design
Compression

PROBLEM AREA 4: FUNCTIONS

"functions and graphs represent one of the earliest points in mathematics at which a student uses one symbolic system to expand and understand another"[27]

SYLLABUS OBJECTIVES (May-June 2018, pp. 30-31):

Section 7: Relations, Functions and Graphs

Students should be able to:

1. Explain basic concepts associated with relations
2. Represent a relation in various ways
3. State the characteristics that define a function
4. Use functional notation
5. Distinguish between a relation and a function
6. Draw graphs of linear functions
7. Determine the intercepts of the graph of linear functions

OBSERVATIONS:

Question 5 of the 2013 CSEC Mathematics Paper 02 tested concepts related to Functions. The average score was "3.00 out of 11"[28]. The report suggested that flow charts may be an effective method for teaching the "composition of functions"[29]. According to the 2014 report, most students attempted to solve the optional question related to functions. However, many experienced difficulties with the algebra involved and as such, were unable to successfully complete the question[30].

[27] Leinhardt, Zaslavsky and Stein, 1990, p. 2
[28] Caribbean Examinations Council, 2013, p. 5
[29] Caribbean Examinations Council, 2013, p. 6
[30] Caribbean Examinations Council, 2014

COMMON ISSUES:

The following are common issues encountered by students when studying Functions:

- Students tend to confuse inverse and composite functions.
- Some students have difficulty substituting numbers into functions.
- When calculating composite functions, students may carry out the substitution in the incorrect order.
- Students struggle to connect graphs with functions.
- Some students confuse the various types of relations, i.e., one-one, one-many, many-one, and many-many.
- Students experience problems generating a frequency table and relating it to ordered pairs and the graph.
- Students are unaware of the myriad of ways that functions are used in various disciplines, be it academic or otherwise.
- Students sometimes lack a proper or effective revision strategy.

TOPIC OVERVIEW:

What are functions?

Relations and Functions as Mathematical concepts have far reaching applications in many spheres of learning. Some of the areas include: Physics, Aeronautics, Chemistry, Biology, Social Studies, Information Technology, Business, Economics, Medicine, Radiology, Refrigeration and Air-conditioning, Converting units and other areas and sub-categories. In light of this, it is surprising that people still ask the question *"What does Mathematics have to do with real life?"*

Relations and Functions is just one topic. Part of the challenge as educators in the field of Mathematics is to allow students to view the landscape of influence that the subject covers. Concept maps help with this metaphorical binocular vision..

Relation:

A ***relation*** is a way of connecting a set of things such as numbers or people[31].

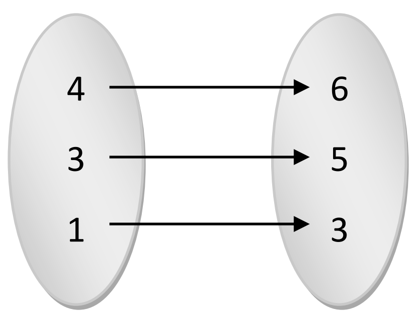
Example: ‘ ____ is the mother of ____’

‘ ____ is the father of ____’

‘ ____ is 5 more than ____’

‘ ____ is greater than ____’

Relations be shown by an ***arrow diagram***.

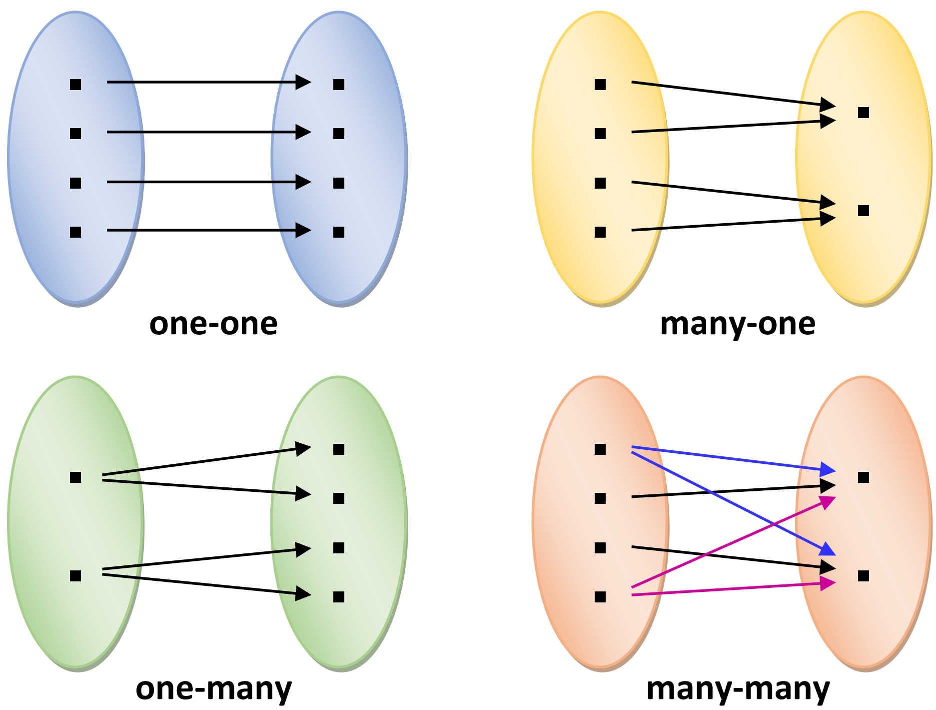


[31] Abdelnoor, 1979, p. 87

If a relation is represented by an arrow diagram, then the ***inverse relation*** is represented by the same arrow diagram but with the direction of the arrows reversed[32].

Relations can also be expressed as ***ordered pairs***: (4, 6), (3, 5), (1, 3). These can be expressed using a Cartesian graph.

Relations can be any one of four types of correspondence: one-one (*read "one to one"*); one-many; many-one; many-many.



Functions:
A ***function*** is a special kind of relation in which each ***object*** is mapped onto only one ***image***. Functions are also known as ***mappings***[33]***.***

A function must be either a **one-one relation** or a **many-one relation**.

[32] Tat Huat, Beng Theam and Gark Kim, 1977, p. 25
[33] Abdelnoor, 1979, p. 40

LESSON 4: UNDERSTANDING FUNCTIONS EXPRESSED IN DIFFERENT WAYS

OBJECTIVES: Students should be able to explain basic concepts related to functions and use a Cartesian graph to draw functions.

MATERIALS AND RESOURCES:

- Graph paper and folder sheets
- Pen, pencil and ruler
- Calculator

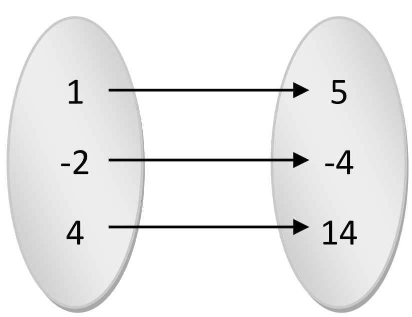
TEACHING TIME: 1 - 1 ½ hours (Double period)

SET INDUCTION:

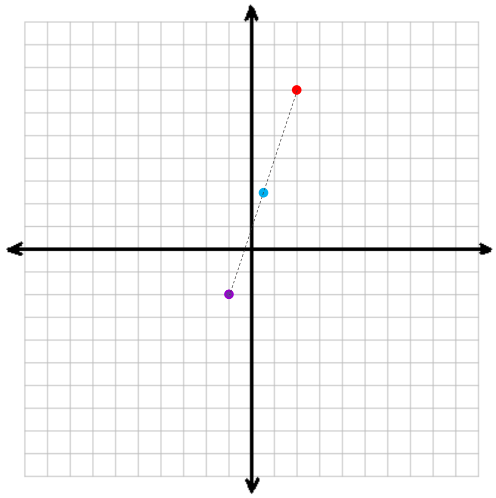
- Allow students to reminisce about a recent football game and the key substitutions that were made by the coach. Recall that for a substitution to be completed a player must leave the field as another enters.
- Compare the substitution in the football game to one that is done when f(x) becomes f(2), f(-3) etc. x leaves and 2 enters etc.

LESSON DEVELOPMENT:

- Define: ***relation***, ***function***, ***domain***, ***codomain***, ***range***
- Show that **f(x) = 3x + 2** is a mapping
  - $f(\mathbf{1}) = 3(\mathbf{1}) + 2 = 5$ $\quad$ $f(\mathbf{-2}) = 3(\mathbf{-2}) + 2 = -6 + 2 = -4$ $\quad$ $f(\mathbf{4}) = 3(\mathbf{4}) + 2 = 12 + 2 = 14$
- Express the mappings as ordered pairs: (**1**, 5), (**-2** -4) and (**4**, 14). Note that the value of the unknown, x, is recorded first.
- Use an arrow diagram to demonstrate the mapping



- Represent the mapping on a graph. When plotted, the ordered points give a straight line



- Introduce composite functions. If **f(x) = 3x + 2** and **g(x) = x^2 + 1** then there can be a composite functions **fg(x)**. Although composite functions are read left to right, when solving, the substitution is done right to left.
  - Assume one must solve fg(2)
  - First find g(2): g(**2**) = 2^2 + 1 = 4 + 1 = **5**
  - Given **g(2) = 5**, f**g(2)** becomes f**(5)**
  - Find f(5): f(**5**) = 3(**5**) + 2 = 15 +2 = **17**
  - Therefore, **fg(2) = 17**
- Show that by using substitution, the composite function can also be found in terms of x
  - fg(x) = f(**g**)
    = f(**x^2 + 1**)
    = 3(**x^2 + 1**) + 2
    = **$3x^2$ + 3** + 2
    = $3x^2$ + 5
- Demonstrate that substituting x = 2 into **fg(x)** in terms of x, produces the same result as previously calculated.
  - fg(**2**) = 3(**2**)^2 + 5
    = 12 + 5
    = 17

- Solve for another composite function **gf(x)**
  - Assume one must solve gf(2)
  - First find f(2): f(**2**) = 3(**2**) + 2 = 6 + 2 = **8**
  - Given **f(2) = 8**, gf(2) becomes g**(8)**
  - Find g(8): g(**8**) = (**8**)^2 + 1 = 64 + 1 = **65**
  - Therefore, **gf(2) = 65**

  - gf(x) = g(**f**)
    = g(**3x + 2**)
    = (**3x + 2**)^2 + 1
    = (**3x + 2**)(**3x + 2**) + 1
    = 9x^2 + 6x + 6x + 4 + 1
    = 9x^2 + 12x + 5

  - gf(**2**) = 9(**2**)^2 + 12(**2**) + 5
    = 36 + 24 + 5
    = 65

- Advise students that both approaches should be learnt:
  - Substitute the given value into the first function and then substitute the output into the second function
  - Substitute the algebraic expression of the first function into the second function to find the expression of the composite function. Then, substitute the given value into the new algebraic expression.
- Emphasize that fg(2) ≠ gf(2)

GLOSSARY:

Codomain: The set to which initial elements are mapped (outputs)

Composite function: A function of a function (nested)

Domain: The initial set of elements or inputs

Function: A function is a special kind of relation in which each object is mapped to **only one image**. Functions are also known as mappings.

Image: The corresponding output or result of the relation from the object

Inverse function: A function that reverses the direction of the mapping produced by a first function. For the inverse function to exist a one-one correspondence must be established.

Mapping: The matching of the elements from one set to the elements of another set using a given rule

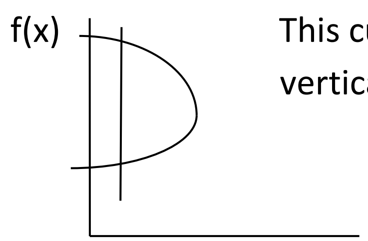
Object: Each element in the domain

Ordered pairs: Bracketed pairs separated by commas that give each domain and codomain. E.g. (2,6)

Range: A subset of the codomain – it is made up of those elements in the codomain that were mapped from an element in the domain

Relation: A relation is a way of connecting a set of things such as numbers or people

Vertical Line Test: A test done on a graphical representation of a relation to determine if it also a function. If a vertical line does not pass through two or more points on the graph, it is a function. The vertical line must pass through one and **only one point** on the graph.



This curve is not a function since the vertical line cuts it in two places

STUDENT EXERCISE:

- Practice substitution in simple algebraic functions
- Apply concept to composite functions
- Draw a graph of simple functions

(1) Given **f(x)** = 3x + 4, **g(x)** = 50 – x^2 and **h(x)** = 3(x + 4), find:

(a) f(2)	(b) g(2)	(c) h(2)	(d) f(0)
(e) g(0)	(f) h(0)	(g) f(7)	(h) g(7)
(i) h(7)	(j) f(-1)	(k) g(-1)	(l) h(-1)

(2) Given **f(x)** = x^2 + x + 3, find:

(a) f(0) (b) f(1) (c) f(2)

(3) Given **g(x)** = $\frac{8x+11}{7}$, find g(-4)

(4) If **m(x)** = $\frac{2x+4}{x-6}$, find:

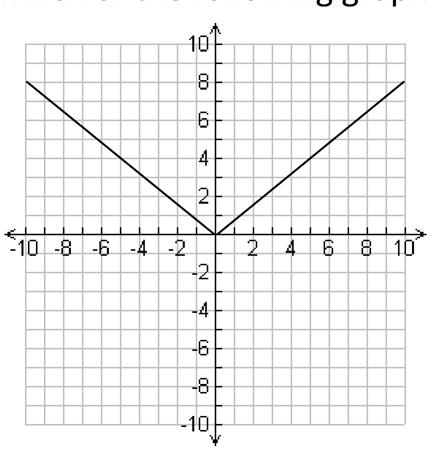
(a) m(7) (b) the value of x for which m(x) = 3

(5) If **f(x)** = x + 3, **g(x)** = 5x and **h(x)** = x – 3, find:

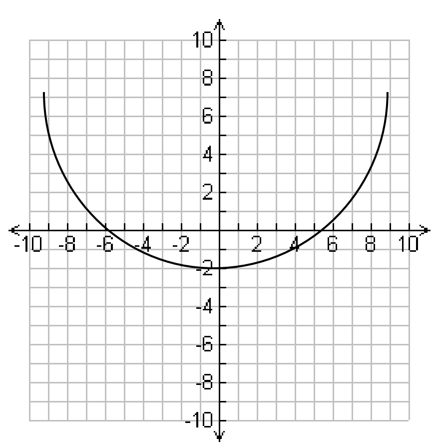
(a) fg(x)	(b) gf(x)	(c) hg(x)	(d) gh(x)
(e) fg(4)	(f) gf(5)	(g) hg(7)	(h) gh(2)

(6) Indicate which of the following graphs are functions and which are not:

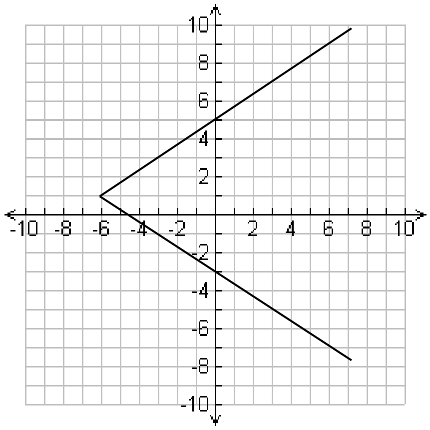
(a)



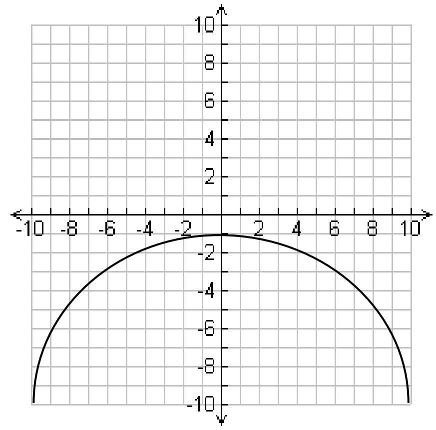
(b)



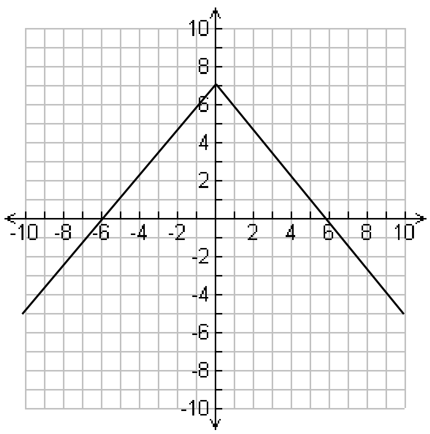
(c)



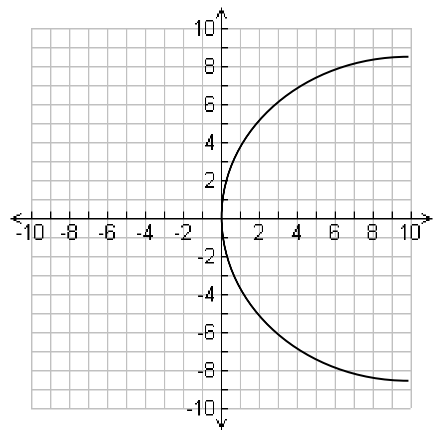
(d)



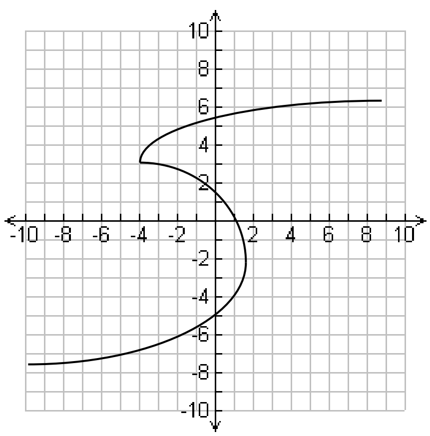
(e)



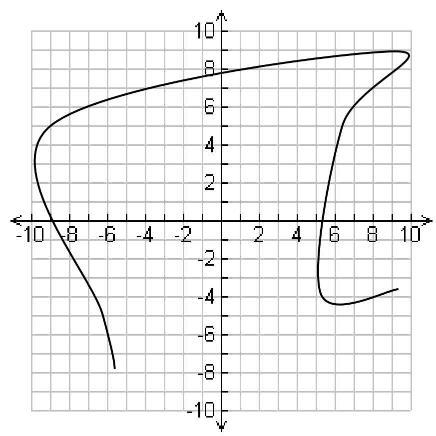
(f)



(g)



(h)



(7) Given the function **f(x)** = 3x + 5,

(a) Complete the following table

x	-1	0	1	2	3
f(x)			8		14

(b) Write the result of the table above as a set of ordered pairs.

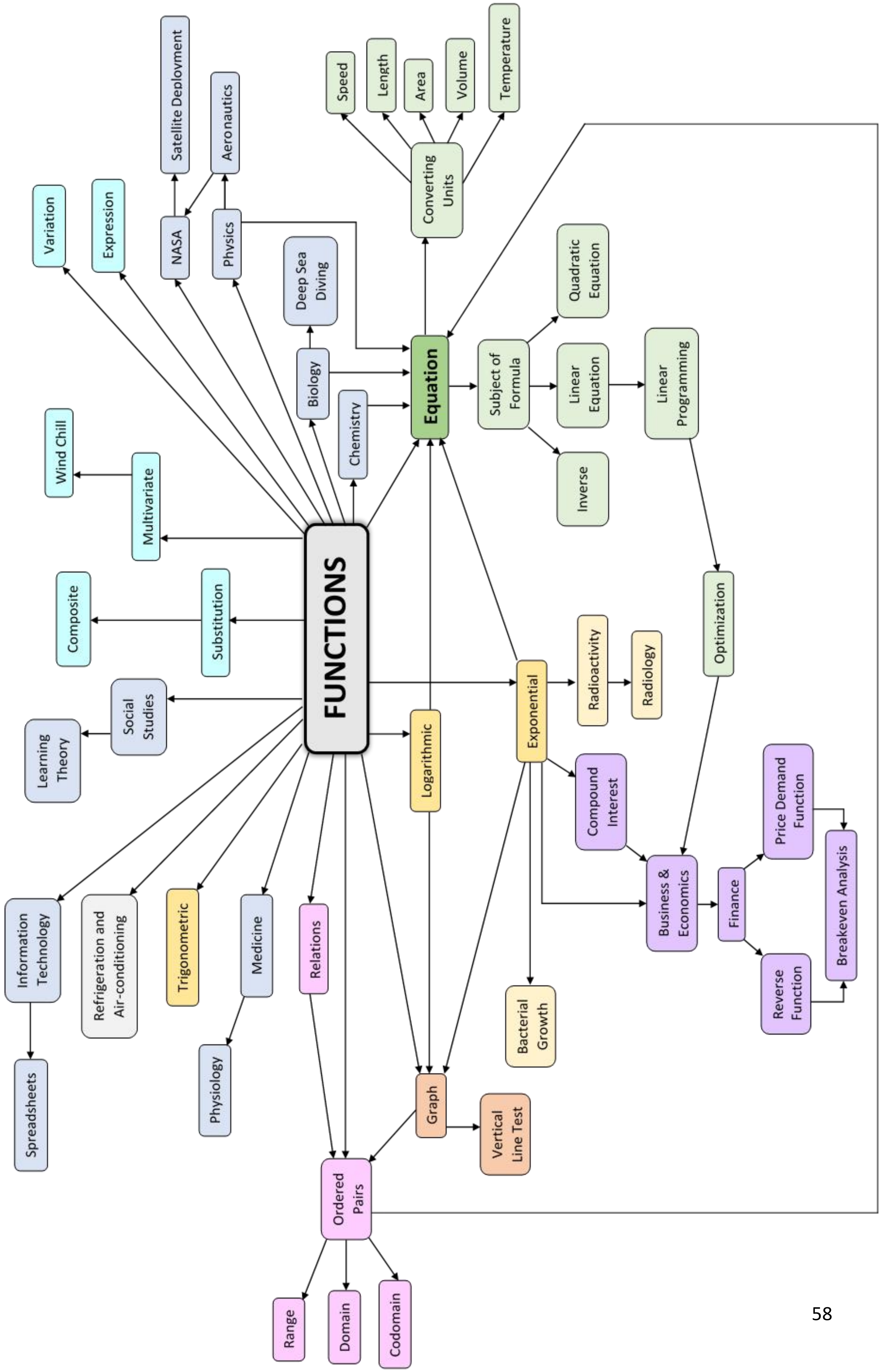
(c) Draw a graph representing the ordered pairs of x (horizontal axis) vs f(x) (vertical axis). Use a scale of 2cm: 1 unit on the horizontal axis and 1cm: 1 unit on the vertical axis. Where does the drawn graph intersect the vertical axis?

EXERCISE SOLUTIONS:

1.	(a) 10 (f) 12 (k) 49	(b) 46 (g) 25 (l) 9	(c) 18 (h) 1	(d) 4 (i) 33	(e) 50 (j) 1
2.	(a) 3	(b) 5	(c) 9		
3.	-3				
4.	(a) 18	(b) 22			
5.	(a) 5x + 3 (f) 40	(b) 5x + 15 (g) 32	(c) 5x – 3 (h) -5	(d) 5x – 15	(e) 23
6.	(a) yes (f) no	(b) yes (g) no	(c) no (h) no	(d) yes	(e) yes
7.	(a) 2, 5, 11	(b) {(-1,2), (0,5), (1,8), (2,11), (3,14)}	(c) 5		

EVALUATION: Assess students' understanding of the concepts through evaluation.

TEACHER'S REFLECTION: Reflect on lesson execution and evaluation results. Determine which concepts were not understood or require further explanation.


FUNCTIONS
Equation
Converting Units
Speed
Length
Area
Volume
Temperature
Subject of Formula
Quadratic Equation
Linear Equation
Linear Programming
Inverse
Optimization
Biology
Deep Sea Diving
Chemistry
Physics
Aeronautics
NASA
Satellite Deployment
Expression
Variation
Multivariate
Wind Chill
Substitution
Composite
Social Studies
Learning Theory
Exponential
Radioactivity
Radiology
Compound Interest
Business & Economics
Finance
Price Demand Function
Reverse Function
Breakeven Analysis
Bacterial Growth
Logarithmic
Graph
Vertical Line Test
Ordered Pairs
Range
Domain
Codomain
Relations
Medicine
Physiology
Trigonometric
Refrigeration and Air-conditioning
Information Technology
Spreadsheets

Problem Area 5: Trigonometry

"Trigonometry presents many first-time challenges for students:
It requires students to relate diagrams of triangles to numerical relationships
and manipulate the symbols involved in such relationships."[34]

SYLLABUS OBJECTIVES (May-June 2018, p. 38):

<u>Section 8: Geometry and Trigonometry</u>

Students should be able to:

1. Use Pythagoras' Theorem to solve problems
2. Define the trigonometric ratios of acute angles in a right triangle
3. Relate objects in the physical world to geometric objects
4. Apply the trigonometric ratios to solve problems
5. Use the sine and cosine rules to solve problems involving triangles
6. Solve problems involving bearings

OBSERVATIONS:

Question 10 of the 2013 and 2014 examination papers included concepts related to Trigonometry. In 2013, "[t]he question was attempted by 44 per cent of the candidates" who on average, earned "2.98 out of 15"[35]. The following year showed a further decline in performance. In 2014, the question "was attempted by 43 per cent of the candidates" with the average score decreasing to "2.7 out of 15"[36].

[34] Weber, 2008, p. 144
[35] Caribbean Examinations Council, 2013, p. 9
[36] Caribbean Examinations Council, 2014, p. 9

COMMON ISSUES:

The following are common issues encountered by students when studying Trigonometry:

- Students exhibit a lack of understanding of the cyclical nature of Trigonometric ratios.

- Students may use the wrong trigonometric ratio in a given question.

- Some students fail to realize that the sine and cosine of angles cannot lie outside the region bounded by +1 and -1.

- Students may use the scientific calculator incorrectly. For example, students may be calculating in radians instead of degrees.

- Students may make rounding errors and skew results if working with 1 or 2 significant figures instead of 3 or 4 significant figures.

- Some students are unable to recognize when the sine rule and cosine rule should be used.

- Some students are unable to recognize and link Pythagoras' Theorem to Trigonometry when needed.

- Students tend to forget that the main assumption behind Trigonometric ratios is a right-angled triangle.

- Students sometimes lack a proper or effective revision strategy.

TOPIC OVERVIEW:

What is Trigonometry?

The name *Trigonometry* is derived from the Greek words meaning “triangle” and “to measure”. It was so called because in its beginnings it was mainly concerned with the problem of “solving the triangle”. This involved finding all the sides and angles of a triangle given some of these were known[37]. In light of this, it is said that:

> “Trigonometry is the study of triangles with regard to their measurements and the relationships between those measurements using ***trigonometric ratios***, and also goes on to deal with ***trigonometric functions***.”[38]

Trigonometry has applications in: Geography: cartography, latitude and longitude, time zones, global positioning systems (GPS); Archaeology; Aviation; Astronomy: calculating distances to stars; Navigation; Physics: electricity, electronics, periodic functions, sound waves, light waves, harmonic motion, high tide and low tide calculations, criminology; Architecture: house construction, roof inclination, structural load calculations; Surveying; Video games; Bearings; Oceanography; Engineering: civil, electrical, sound, mechanical, marine; Information Technology; Natural Sciences. Clearly, Trigonometry has extensive use in many facets of life.

Technology has allowed cellular phones to be used as instruments of measurement in Trigonometry. There are phone applications (Apps) that give angles of elevation and depression. Some Apps even give bearings. Nevertheless, the concepts must still be learnt. Hiking compasses help users to follow directions and chart a course. The electronic calculator seems like old technology now but less than fifty years ago, 3 and 4 figure tables had to be used to find the sine, cosine and tangent of angles and their inverses. The slide rule was common in that era as well. This of course is a reminder that there is history in Mathematics apart from it being a Language as well. This history spans centuries and millennia and is quite intriguing.

[37] Abbot, 1970
[38] Tapson, 1999, p. 146

Figure 8 shows a hiking compass that allows bearings to be measured, in addition to identifying the cardinal points (North, South, East and West). It also has an angle of elevation attachment. A simple device like this can save a person's life in the forest since no battery or recharging is needed. Men of the military depend on this as well and it is part of the toolkit used for conducting military exercises. Of course, any large metal objects that are close by can influence measurement negatively.

Figure 8: A Hiking Compass

The shadow stick is simply a stick in sunlight casting a shadow on level ground. The stick (a metre rule for example), the shadow (on level ground) and a ray of sunlight together form a right angle triangle for a particular elevation of the sun and its light rays. Figure 9 shows Ervin, a primary school student, beginning to measure a shadow of a metre rule that Thia is holding upright.

Figure 9: Using a Shadow Stick

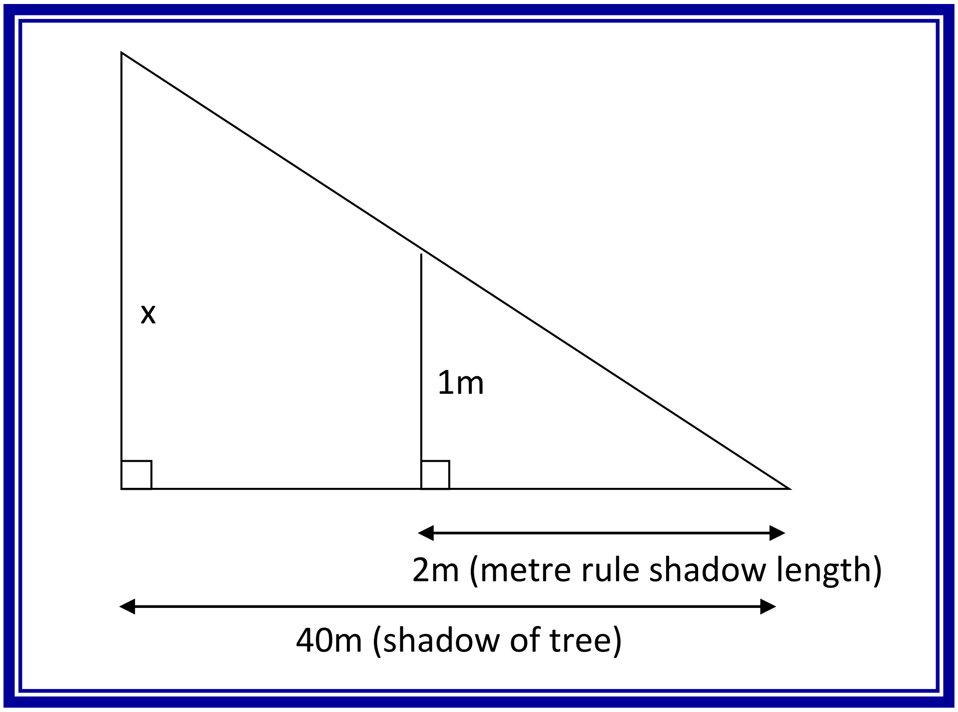


Figure 10

A tree of height **x** is shown in the diagram. It casts a shadow of 40m. The diagram is not drawn to scale. By the ratio of sides of similar triangles:

$$x : 40 = 1 : 2 \qquad \text{or} \qquad \frac{x}{40} = \frac{1}{2}$$

Solving this equation gives x = 20m.

The shadow of the tree and the shadow of the metre rule need not be connected as in the diagram. The only connection needed is that the measurements are taken at the same time when the angle of the sun is the same for both of them.

This ratio of sides given by the stick and its shadow is called the ***tangent*** (tan) of the given angle (the elevation of the sun).

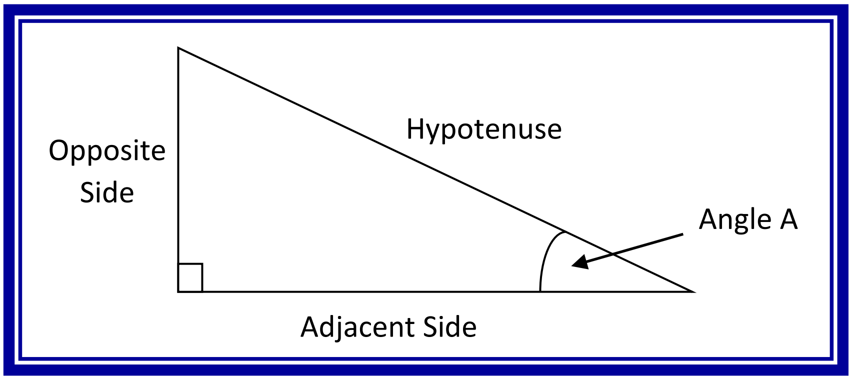


Figure 11

$$\text{The tangent of } \hat{A}, \textbf{tan } \hat{\mathbf{A}}, = \frac{opposite}{adjacent}$$

By similar triangles this ratio of sides is fixed for any given angle. It will change as the angle changes.

$$\text{The cosine of } \hat{A}, \textbf{cos } \hat{\mathbf{A}} = \frac{adjacent}{hypotenuse}$$

This ratio is also fixed for any given angle. It will change as the angle changes.

$$\text{The sine of } \hat{A}, \textbf{sin } \hat{\mathbf{A}} = \frac{opposite}{hypotenuse}$$

Yet again, this ratio is fixed for any given angle. It will change as the angle changes.

These ratios can be remembered by the acronym **SOHCAHTOA** *(pronounced "Soak ah toe..ahh")*

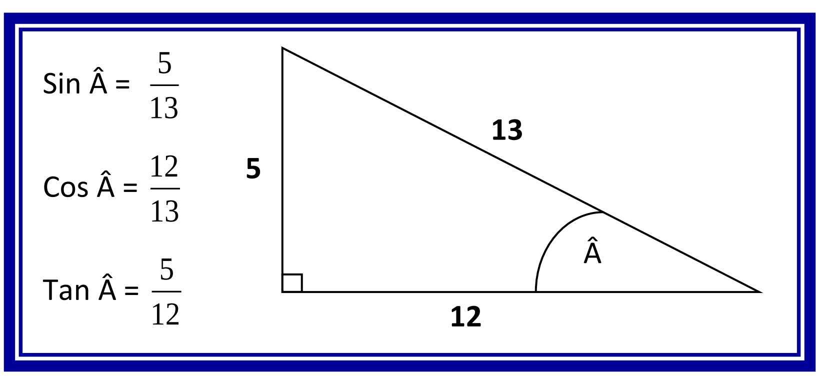


Figure 12

Figure 13: Thia is having a good “Soak ah toe…ahhh”

SOH: ***S***ine of a given angle is ***O***pposite side divided by ***H***ypotenuse
CAH: ***C***osine of a given angle is ***A***djacent side divided by ***H***ypotenuse
TOA: ***T***angent of a given angle is ***O***pposite side divided by ***A***djacent side
Students cannot, **or should not**, forget it now!

The values of these trigonometric ratios were tabulated from ancient times to develop a table (reference) of results. A high degree of accuracy was made possible by using many Geometric theorems and principles.

A simple method, to about two significant figures accuracy, is illustrated on the next page for students to replicate and better understand how the ratios connect to numbers. A method of great accuracy, for select angles only, is illustrated on pages 66 and 67.

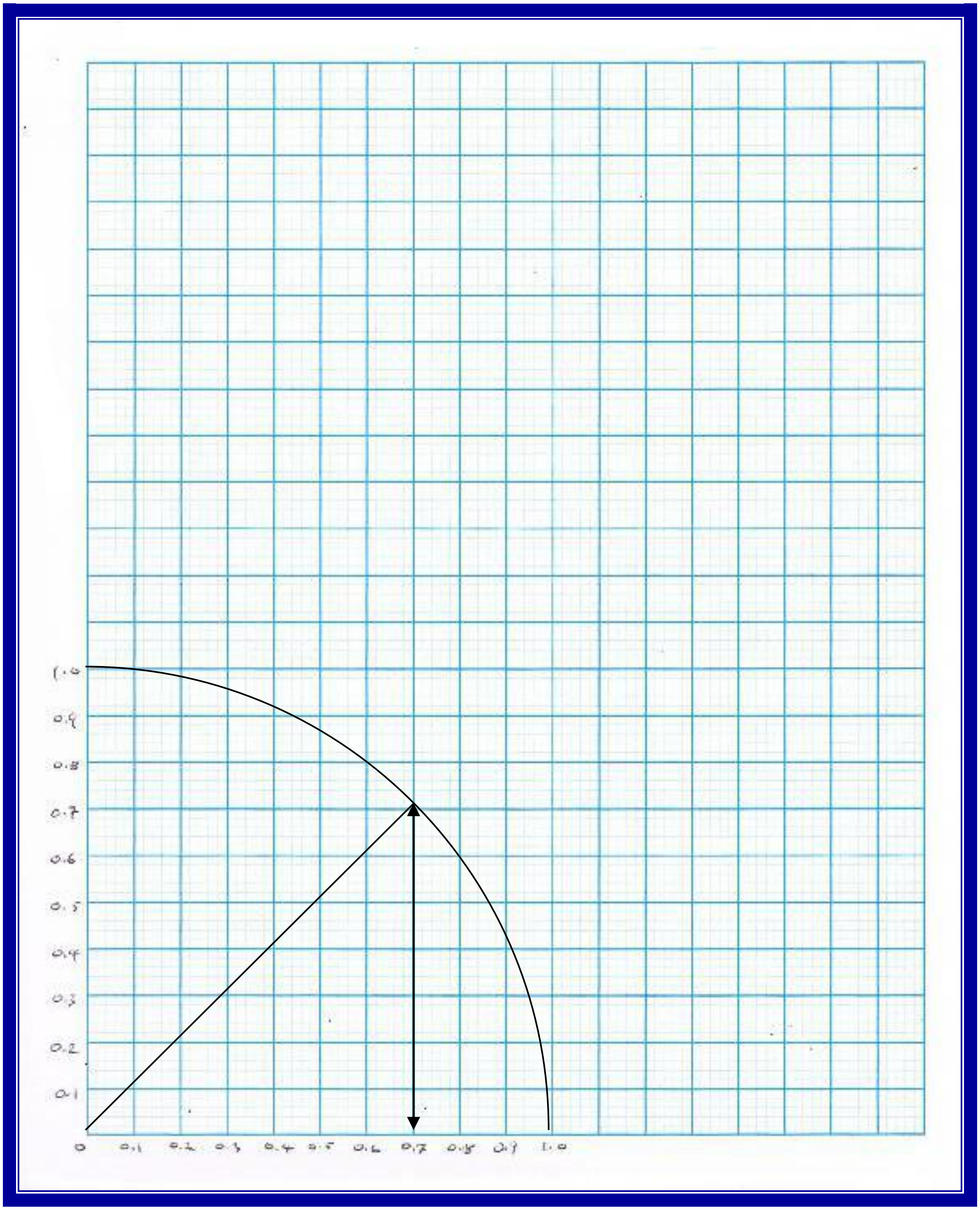


Figure 14

The diagonal line drawn in the figure above is at an angle of 45^0 to the horizontal line and has a length of 1 unit. Now find: sin 45^0, cos 45^0, tan 45^0

From the graph: sin $45^0 = \frac{0.7}{1.0}$ cos $45^0 = \frac{0.7}{1.0}$ tan $45^0 = \frac{0.7}{0.7}$

Therefore: sin $45^0 =$ 0.7 cos $45^0 = 0.7$ tan $45^0 =$ 1

Lines at 10^0, 20^0, 30^0, 40^0, 50^0, 60^0, 70^0, 80^0 can now be drawn (using protractors) starting at the origin O and stopping at the point of meeting the arc drawn. For each of these lines, a vertical line can be drawn from the meeting points at the arc down to the horizontal axis. These lines will allow students to determine the sine, cosine and tangents for all the angles represented, though not to a high degree of accuracy. The level of accuracy can be increased using other geometric methods that allow users to tabulate the sine, cosine and tangents of given angles.

The diagram illustrated below is derived using Pythagoras' Theorem.

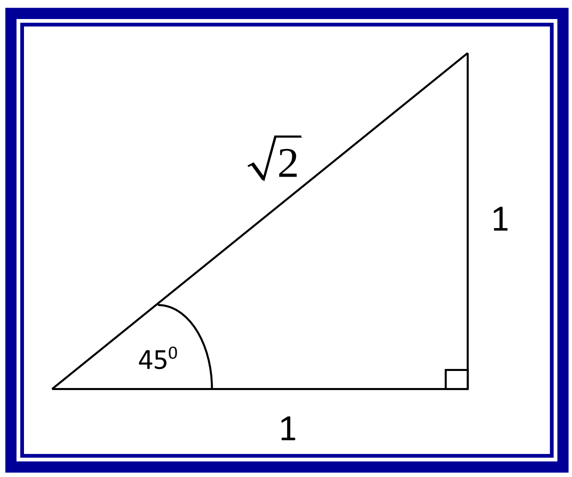


Figure 15

The 45^0 angle is formed since it is an isosceles triangle with a 90^0 angle (right angle).

Students can now calculate sin $45^0 = \frac{1}{\sqrt{2}}$, cos $45^0 = \frac{1}{\sqrt{2}}$ and tan $45^0 = 1$ (exactly).

These latter calculations are all accurate depictions of the ratios. The geometrical illustrations are just two ways of getting values of sine, cosine and tangent that can be used to develop a table of values. Other methods provide results to 3 and 4 significant figures. However, in this present era, scientific calculators can be used to determine results with even greater accuracy.

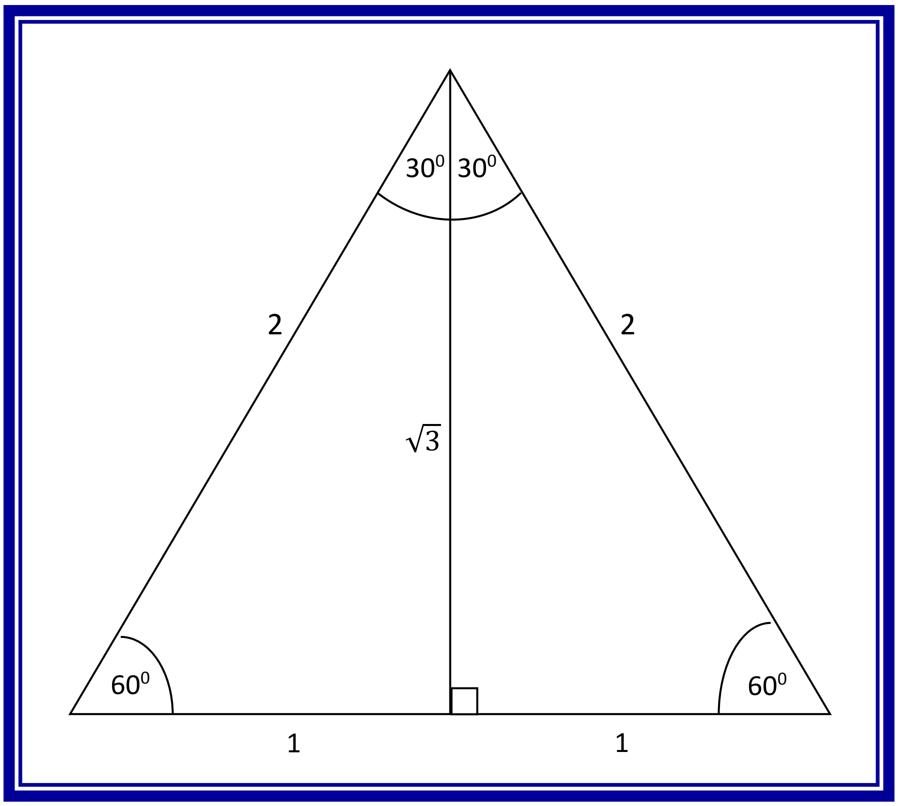


Figure 16

An equilateral triangle of side 2 units can be subdivided into two right angled triangles as shown above. The base is divided into two equal lengths of side 1 unit and the vertical line has height $\sqrt{3}$ by virtue of Pythagoras' Theorem.

So $\sin 60^0 = \frac{\sqrt{3}}{2}$ $\cos 60^0 = \frac{1}{2}$ $\tan 60^0 = \sqrt{3}$

and $\sin 30^0 = \frac{1}{2}$ $\cos 30^0 = \frac{\sqrt{3}}{2}$ $\tan 30^0 = \frac{1}{\sqrt{3}}$

These are all precise values of the Trigonometric ratios

These common precise values are tabulated below.

	0^0	30^0	45^0	60^0	90^0
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

LESSON 5: EXPLORING TRIGONOMETRIC RATIOS

OBJECTIVES: Students should be able to perform basic calculations and use a graphical approach to calculate sine, cosine and tangent of selected angles.

MATERIALS AND RESOURCES:

- Graph paper and folder sheets
- Pen, pencil and ruler
- Geometrical instruments such as protractor and compass
- Calculator

TEACHING TIME: 1 - 1 ½ hours (Double period)

SET INDUCTION:

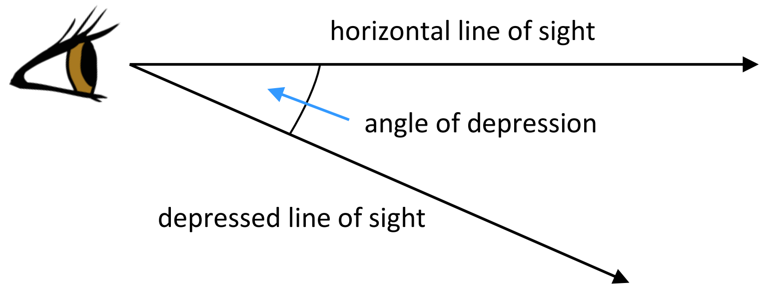
- Using a calculator, demonstrate how to find sin 40^0, cos 50^0 etc. Allow students to do the same and reproduce identical answers.
- The teacher should demonstrate the use of the DRG key and show that the calculator toggles between three different angle units.

LESSON DEVELOPMENT:

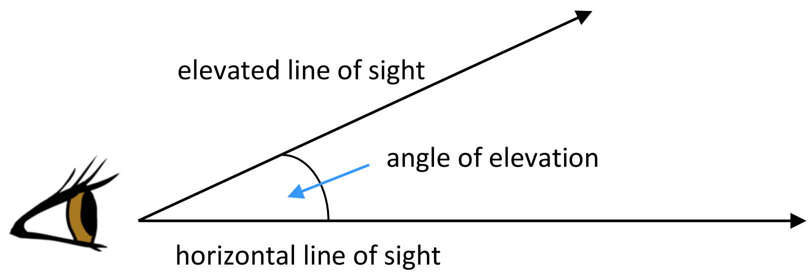
- On a new graph sheet, instruct students to reproduce Figure 14 with its units labelled, the arc constructed, the diagonal line that meets the arc and the vertical arrowed line that drops from that point.
- Illustrate the tangent calculation of 45^0. Do a measured sin 45^0 and cos 45^0.
- Ask students to draw lines measured in 10^0 increments up to 80^0 starting from the origin and ending at the arc of radius 1 unit.
- For each line meeting the arc, draw a vertical line downward with an arrow touching the horizontal axis.
- Demonstrate to students how to find sin, cos and tan of all the given angles graphically.

GLOSSARY:

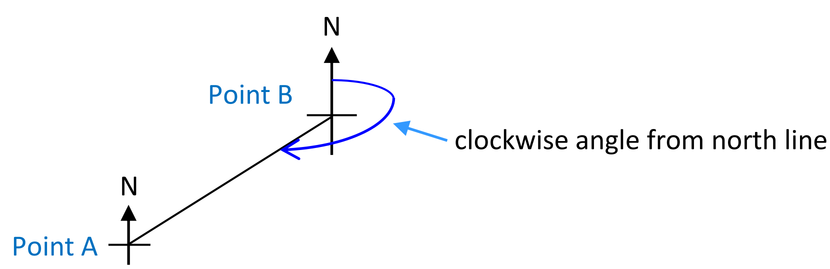
Adjacent side:	This is the side **next to** the given angle that is not the hypotenuse in a right angled triangle.
Angle of depression:	The angle through which a horizontal line of sight is rotated downward to a depressed line of sight.



Angle of elevation:	The angle through which a horizontal line of sight is rotated up to an elevated line of sight.



Angle:	A measure of turn about a fixed point.
Bearing:	The bearing of a point A <u>from</u> another point B is the clockwise direction of A from a north line taken at B



Cosine of an angle:	The ratio of the adjacent side of a right angled triangle to the hypotenuse.

Degree: A unit of measurement of an angle in which a complete revolution is 360 degrees.

Grade: A unit of measure of an angle in which a complete revolution is 400 grades and quarter revolution is 100 grades (a metric type unit).

Hypotenuse: In any right angled triangle the hypotenuse is the longest side and is always opposite to the right angle.

Line of sight: An imaginary line drawn from an eye to an object some distance away.

Opposite side: This is the side that is **opposite** to the given angle in a right angled triangle.

Radian: A unit of measurement of an angle in which 1 radian sweeps out the arc of a circle that is the length of the radius of the circle. A complete revolution is 2π radians.

Ratio: If two quantities are of the same kind, they have a ratio and the ratio of the first to the second is the quotient obtained by dividing the first by the second, whether the quotient be integral or fractional[39].

Right angled triangle: A triangle that has a 90^0 angle as one of its angles.

Similar triangles: Triangles that are shaped alike but different in size. These triangles have the same angles and the respective sides are in the same ratio to each other.

Sine of an angle: The ratio of the opposite side of a right angled triangle to the hypotenuse.

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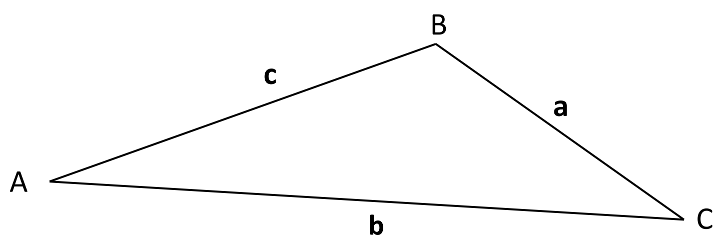
[39] Baker and Bourne, 1904

SOHCAHTOA: An acronym that aids in remembering the basic Trigonometric ratios of sine, cosine and tangent (S=O/H, C=A/H, T=O/A)

Tangent of an angle: The ratio of the opposite side of a right angled triangle to the adjacent side.

RULES:



For the triangle drawn, vertex A will have Â next to it and side "a" opposite. Likewise vertex B and vertex C will have corresponding angles and sides.

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

This is applicable for any triangle whether it has a right angle or not.

cosine rule: $a^2 = b^2 + c^2 - 2bc \cos \hat{A}$ or $b^2 = a^2 + c^2 - 2ac \cos \hat{B}$ or

$c^2 = a^2 + b^2 - 2ab \cos \hat{C}$

STUDENT EXERCISE:

- Measure from graph the sine, cos and tan of the angles 10^0, 20^0, 30^0, ...80^0
- Compare those answers with those found on the calculator.
- Discuss errors and accuracy

(1) Using the graph drawn earlier, illustrate:

(a) Sin 35^0 (b) Sin 48^0 (c) Cos 54^0 (d) Cos 67^0

(2) Using the graph drawn earlier, illustrate:

(a) Tan 25^0 (b) Tan 73^0

(3) Use the calculator to find (to 3 s.f.):

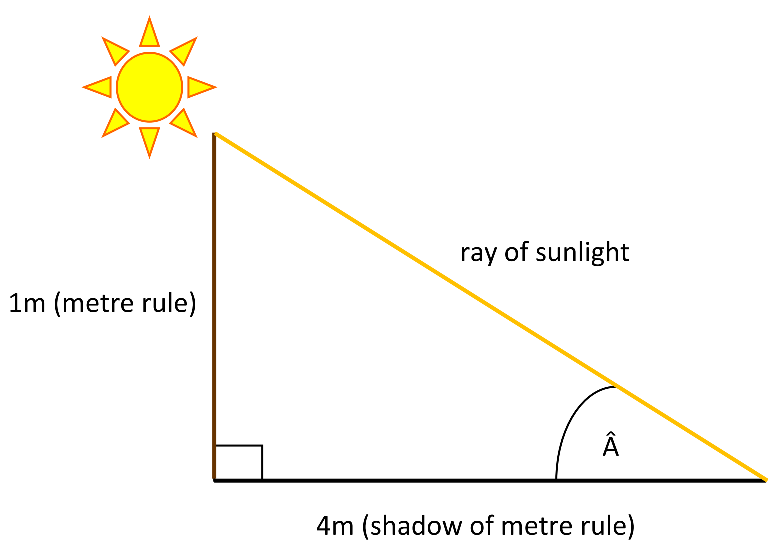
(a) Sin 63^0 (b) Cos 44^0 (c) Tan 85^0 (d) Sin 37^0

(4) Use the calculator to find the following inverse trigonometric ratios (to 3 s.f.):

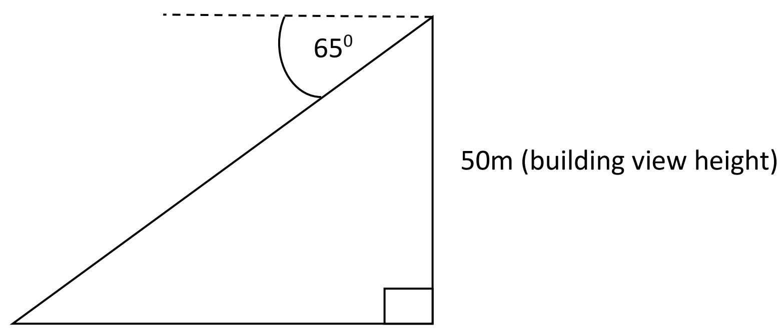
(a) Sin^{-1} 0.5 (b) Tan^{-1} 1 (c) Cos^{-1} 0.5

(5) Compare the results of Question 4 with the table of precise values on page 69.

(6) Find the angle of elevation of the sun, Â, in the following diagram. (Hint: An inverse trigonometric function will be needed).

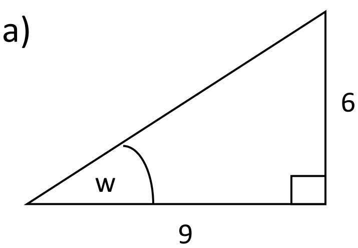


(7) Darlene measures the angle of depression $\hat{D}$ from a building to a car below as 65^o. She knows that her position in the building is 50m above the ground level. How far is the car from the foot (bottom) of the building?

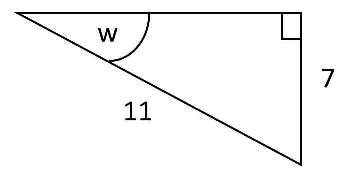


(8) Find the angle marked "w" in each of the following:

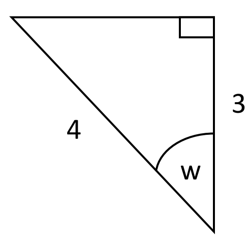
(a)



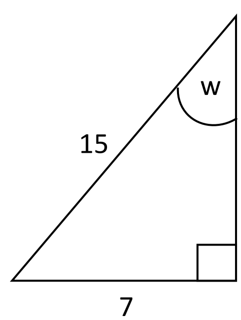
(b)



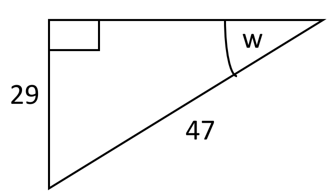
(c)



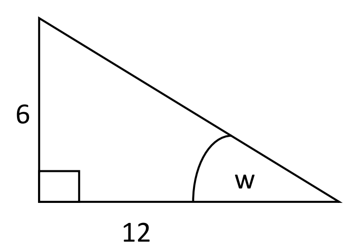
(d)



(e)

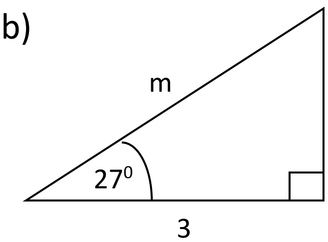


(f)

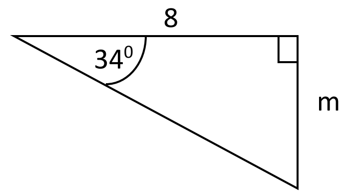


(9) Find the side marked "m" in each of the following

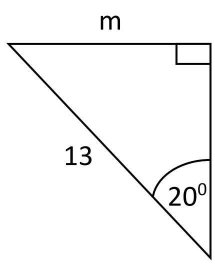
(b)



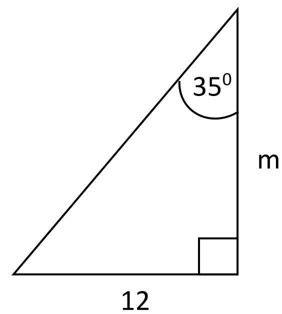
(b)



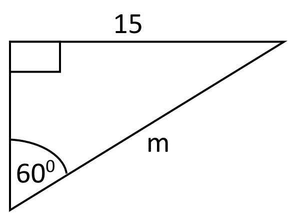
(c)



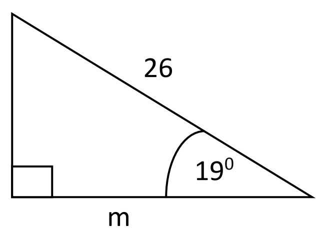
(d)



(e)



(f)

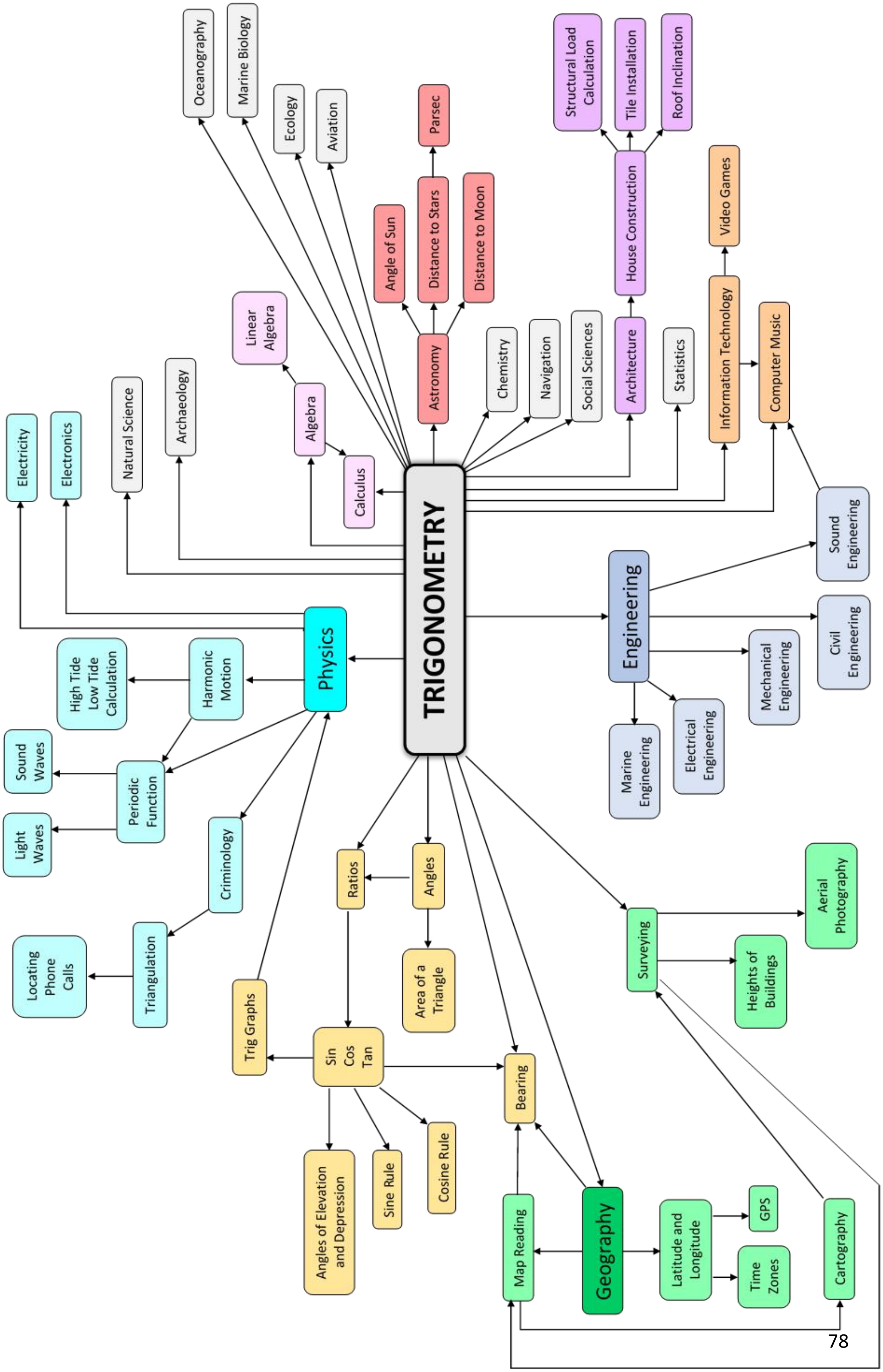


EXERCISE SOLUTIONS:

1.	(a) 0.57	(b) 0.74	(c) 0.59	(d) 0.39	
2.	(a) 0.47	(b) 3.3			
3.	(a) 0.891	(b) 0.719	(c) 11.4	(d) 0.602	
4.	(a) 30^0	(b) 45^0	(c) 60^0		
5.	Observation and discussion question				
6.	(a) 14.0^0				
7.	(a) 23.3m				
8.	(a) 33.7^0 (f) 26.6^0	(b) 39.5^0	(c) 41.4^0	(d) 27.8^0	(e) 51.9^0
9.	(a) 3.4 (f) 24.6	(b) 5.4	(c) 12.2	(d) 17.1	(e) 17.3

EVALUATION: Assess students' understanding of the concepts through evaluation.

TEACHER'S REFLECTION: Reflect on lesson execution and evaluation results. Determine which concepts were not understood or require further explanation.



TRIGONOMETRY
Physics
Electricity
Electronics
Harmonic Motion
High Tide Low Tide Calculation
Periodic Function
Sound Waves
Light Waves
Criminology
Triangulation
Locating Phone Calls
Natural Science
Archaeology
Algebra
Linear Algebra
Calculus
Oceanography
Marine Biology
Ecology
Aviation
Astronomy
Angle of Sun
Distance to Stars
Parsec
Distance to Moon
Chemistry
Navigation
Social Sciences
Architecture
House Construction
Structural Load Calculation
Tile Installation
Roof Inclination
Statistics
Information Technology
Video Games
Computer Music
Engineering
Sound Engineering
Civil Engineering
Mechanical Engineering
Electrical Engineering
Marine Engineering
Ratios
Angles
Area of a Triangle
Sin Cos Tan
Trig Graphs
Angles of Elevation and Depression
Sine Rule
Cosine Rule
Bearing
Surveying
Aerial Photography
Heights of Buildings
Geography
Map Reading
Latitude and Longitude
GPS
Time Zones
Cartography

ADVANCED ACTIVITIES

These activities may be conducted with upper level students as a reinforcement lesson or revision exercise. These activities demonstrate the relationship between Measurement and Trigonometry and allow students to become actively involved in their learning.

ACTIVITY 1: DETERMINE THE HEIGHT OF A TREE (OR BUILDING)

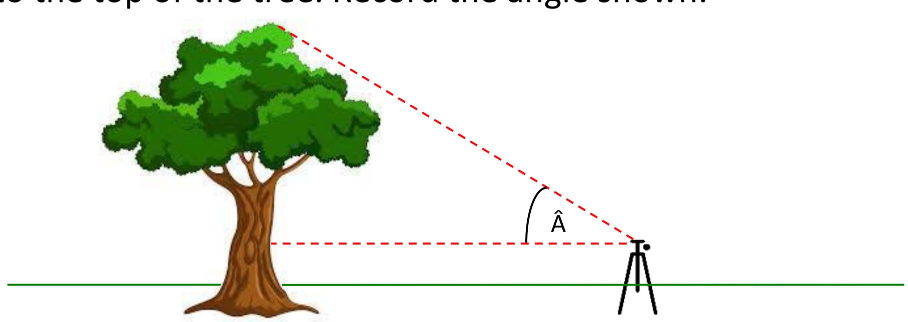
OBJECTIVES: Students should be able to determine the height of a tree (or building) using angles of elevation and measurements.

MATERIALS AND RESOURCES:

- Trundle wheel
- Measuring tape / Metre rule
- Compass on tripod stand
- Pen and paper
- Calculator

GUIDELINES:

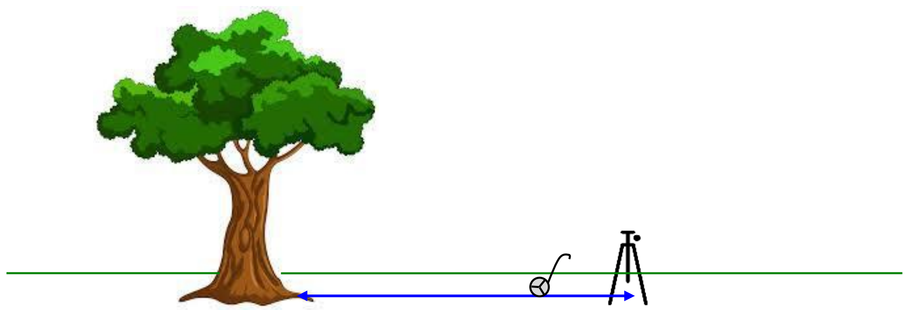
- Place the tripod[40] stand and compass in line with the tree[41] (or building) to be measured.
- Ask students to use the compass find the angle of elevation from the tripod stand to the top of the tree. Record the angle shown.



[40] Icon Archive [Online Image], 2014

[41] Cliparting [Online Image], n.d.

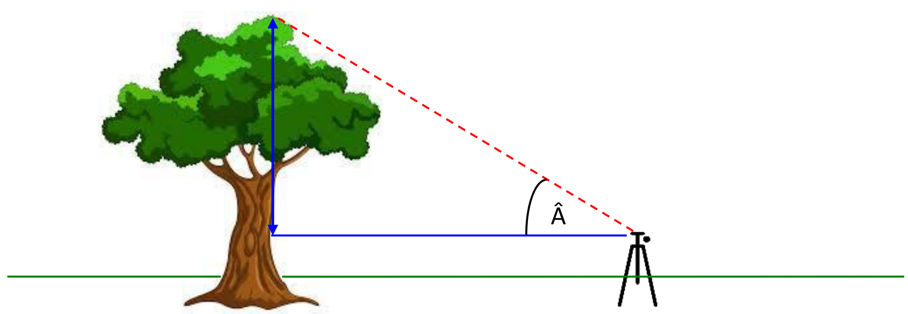
- Ask students to use a trundle wheel to determine the distance between the tripod stand and the tree. Record the measurement found.



- Given the angle of elevation and the length of the adjacent side, the distance from the top of the compass to the top of the tree can be determined using trigonometric ratios.

$$\tan \hat{A} = \text{Opposite} \div \text{Adjacent}$$

$$\text{Opposite} = \tan \hat{A} \times \text{Adjacent}$$



- Use the metre rule or measuring tape to determine the height of the tripod stand. Add this measurement to the length of the ‘opposite’ side to determine the height of the tree.

ACTIVITY 2: FINDING THE HYPOTENUSE GIVEN A SIDE AND AN ANGLE

For this activity, teachers may consider inviting a professional surveyor to participate in the activity. This will allow students to make connections to careers where mathematical skills are often used.

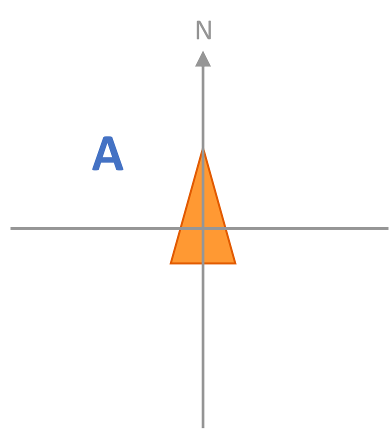
OBJECTIVES: Students should become familiar with finding bearings, angles and measurements.

MATERIALS AND RESOURCES:

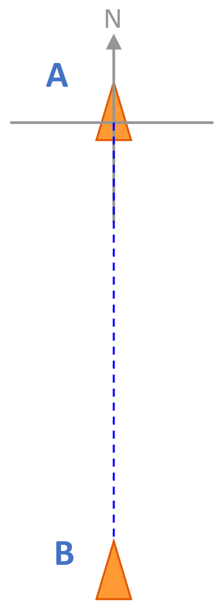
- Trundle wheel
- Compass on tripod stand
- Three cones
- Pen, paper, calculator

GUIDELINES:

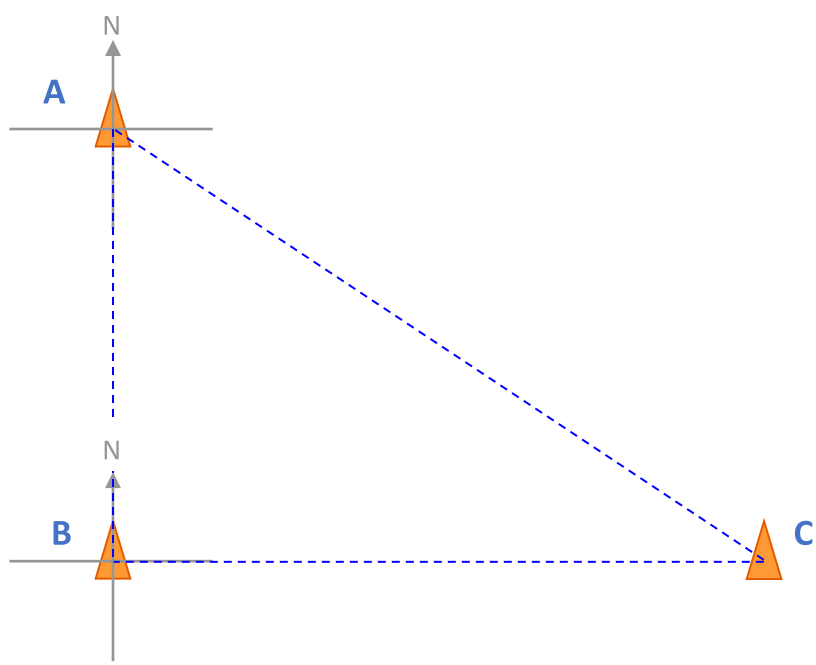
- Place a cone near to the edge of an open field. This will serve as the starting point for the triangle. Label this point 'A'.



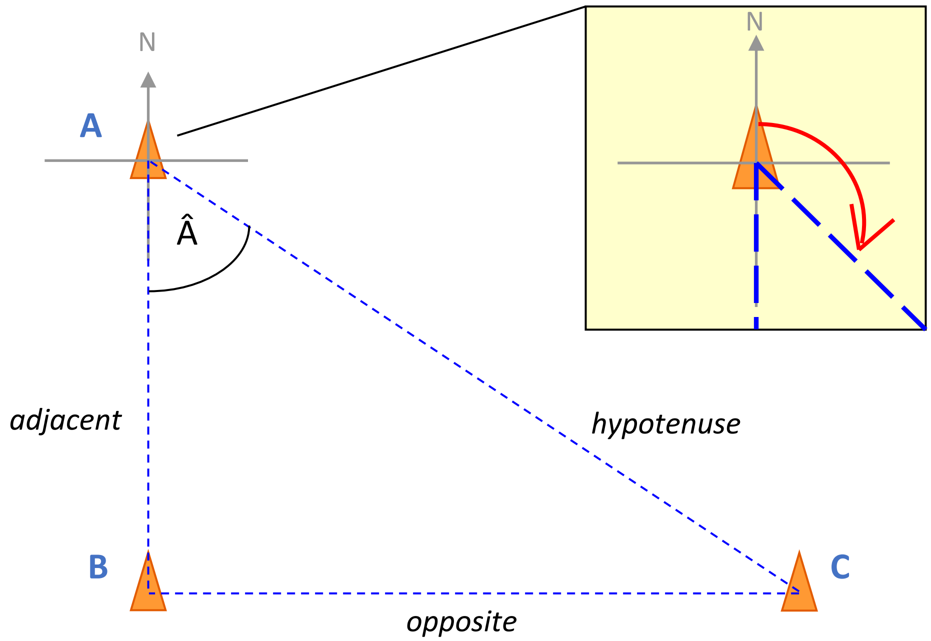
- To form the base of the triangle, use a compass to measure a bearing of 180^0 south from the starting point 'A'. Use another cone to mark the second point, point 'B'.



- Use a trundle wheel to determine the length of line AB. In this example, AB is the **adjacent** side.
- Using the compass, measure a 90^0 bearing east from point 'B'. Use another cone to mark the third point, point 'C'. This forms the **opposite** side, line BC.



- Return to point A. Using the compass, determine the bearing of 'C' from 'A'.



- Subtract the bearing of 'C' from 'A' from 180^0 to find Â.
- Given the length of the adjacent side, as well as Â, the following may be used to determine the length of the hypotenuse, AC:

cos Â = Adjacent ÷ Hypotenuse

Hypotenuse = Adjacent ÷ cos Â

- Use the trundle wheel to find the length of the line AC. Compare the results to the length of the hypotenuse found using the calculations.

FURTHER READING

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4. Website: www.mymaths.co.uk (*Annual subscription by school required)*

5. Website: www.khanacademy.org

6. Website: www.math-only-math.com

7. Website: www.mathforum.org

8. Website: www.nctm.org

9. Website: www.GoogleEarthPro.com or www.GoogleEarth.com

10. Website: www.Edmodo.com

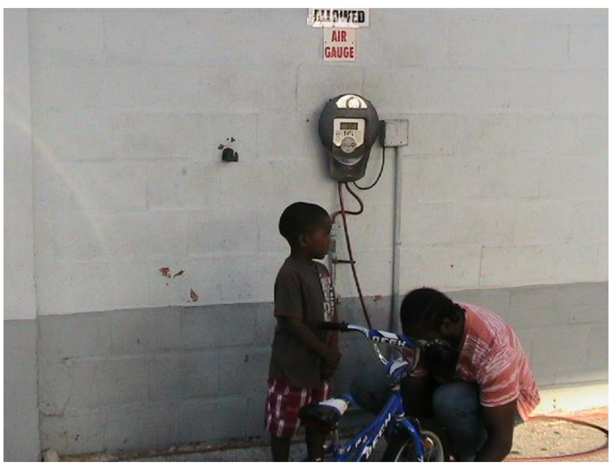
11. Website: www.quizziz.com

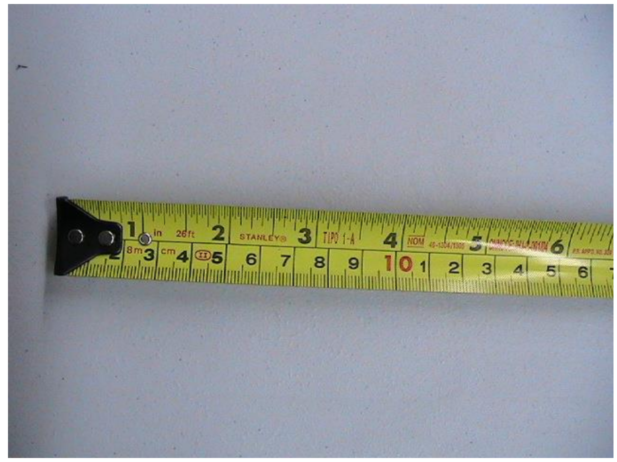
12. Website: www.classmarker.com

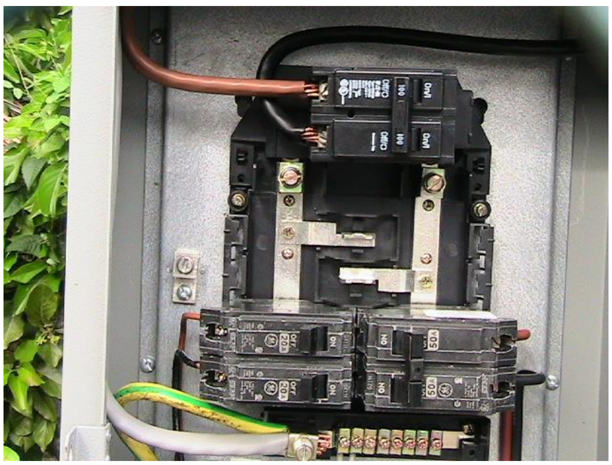
13. Website: www.goformative.com

FURTHER THOUGHTS

Where is the Mathematics in these pictures?







Drama and poetry were instruments of teaching long before the internet and PowerPoint came into being. The internet is less than 25 years old. The use of cellphones also rapidly increased within this same timeframe. How was life before these items of technology?

A UWI Dip. Ed. character many years ago, by the name "General Maths" or "Super Maths" once said,

"Measure Big, measure Small

Imperial, metric and all

With Mathematics you can stand tall

The General say passing the exam is OK

But what is important too

Is making this subject of use to you

The moral of this is plain to see

That the study of Maths is necessary"

(Excerpt from a play done by the UWI Mathematics Dip Ed. Class of 1994-1995)

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