Mathematics Curriculum Support Manual for Primary Schools

Curriculum Support Manual

Rationale

Numeracy, an essential skill for everyone (National Numeracy For Everyone, For Life, 2014), is defined as the use of mathematics involving number and number systems, to understand and interpret information from data, charts and diagrams; to reason in order to solve problems, check answers, understand and explain solutions and make decisions as a result of logical thought. Numeracy however is not limited to number alone, as numeracy skills also include topics related to shape, space and measurement.

In order to be successful at Numeracy, students must have a positive attitude. They must be willing to solve problems involving number, data and measurement. Positive attitudes encourage the user to not give up at the first failed attempt, but to persist. Numeracy skills amongst students can be improved with encouragement and determination. Numeracy levels also increase with self-confidence as the learner improves his/her skills with the use of numbers.

In the exploration of numbers, number patterns and relationships, reasoning skills are developed where generalizations are applied to help solve real life problems. Opportunities are provided for interaction with numbers through the operations and calculations for the purpose of solving mathematical problems. The ability to apply the number operations of addition, subtraction, multiplication and division when calculating, develops skills which include: the ability to provide a realistic estimate, making a reasonable guess, applying and checking answers for reasonableness and accuracy. Logical thinking is promoted as well as the ability to communicate what was done and why a particular method was applied.

Measurement is reflected through the use of numbers to indicate the size or amount of something. Its application in real life, includes but is not limited to the measure of: area, capacity, volume, linear, perimeter, speed, and mass, calendar time, age and duration of time. A working knowledge of arbitrary and standard units is necessary. The ability to measure and interpret numbers, use and read the various measuring instruments and devices are also strengthened.

This booklet is aimed at improving numeracy skills in students in the classroom. Numeracy skills can be actively taught by teachers through careful planning, conscious effort and effective teaching.

Problem Area #1 Math Anxiety

Math Anxiety is often misunderstood and dismissed as being a real condition, this leads to an underestimation of its impact on both young learners and adults. It is a genuine condition which hinders a person's ability to calculate and solve problems with speed and accuracy. Recognizing that this anxiety can be easily transferred to students is a crucial step in stopping the perpetuation of a vicious cycle and is necessary to be able to foster positive attitudes towards mathematics and numeracy skills.

The new Primary Mathematics Curriculum aims to reduce math anxiety and ease the transition from Primary to Secondary school "through the development of appropriate dispositions that would facilitate life-long learning and higher order thinking skills" (Pg.5). Reference is made to "Adding It Up: Helping Children Learn Mathematics" (2001), where it is argued that in developing mathematical proficiency, instructional programs need to include methodology which develops strategic competence in formulating, representing and solving mathematical problems and adaptive reasoning on logical thought, reflection, explanation and justification.

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Problem Area #2 - Problem Solving

Several word problems are often presented at the end of a lesson or chapter, requiring the exact procedure or operation taught in the lesson to be performed. After solving a few of those problems, children automatically repeat the same procedure without much thought or analysis of the problem presented. A genuine mathematical problem is a situation requiring a procedure for solving that which is unknown to the persons attempting to solve. It ceases to be a mathematical problem if the procedure is known beforehand, where instead it is an exercise to be solved.

In the classroom, heavy reliance may be placed on problems involving only one step. Problems which are based on real life experiences and situations generally involve more than one step (multi-step). Attempts should be made to write problems that are close to real life situations.

Problem Area #3 – Recreational Mathematics

There is the misconception that using puzzles and mathematical games in the classroom is only useful after the end of term tests, when students have a bit of free time. Some believe that classroom time is better spent on teaching mathematical concepts, algorithms, exercises, drills and problem solving in preparation for a high-stakes exam.

Certainly, with a curriculum mandated to be followed and a goal of preparing students for a standardized summative test, significant amounts of teaching time is spent on formal learning. Teachers should not easily dismiss mathematical games, for there is educational benefit from exposing students to puzzles and games as they help improve reasoning skills and problem solving abilities in a non-threatening way.

Problem Area #4 – Multiplication Tables and Strategies

Rote learning of the multiplication tables, through the repetition of multiplication facts, has been challenging for students, especially when there is a lack of sufficient opportunity to explore the effect of repeated addition. This leads to an inadequate understanding of the building of the multiplication tables. This is further compounded when the multiplication facts are presented out of context or in isolation as rote learning does not work for all students.

Some students need a variety of strategies, for example skip counting, to help in the recollection of multiplication facts. Students benefit from exercises and activities which help to make connections between multiplication and repeated addition. Activities that encourage students to develop their own meaning of the multiplication facts help students with recall and application of the tables when problem solving.

Problem Area #5 – Elapsed Time – Strategies and Resources

Solving problems involving elapsed time has proven to be challenging for students. The concept of time, the passage of time and duration of time can sometimes be abstract concepts to students especially if they have limited exposure to digital or analog clocks at home or school. Society's trend of reduced dependency on wrist watches and greater reliance on the digital time from cell phones can also impact negatively on students' ability to tell time.

Teaching time in the classroom, may be too abstract as lessons typically focus on telling time and less on understanding the passage of time. In the classroom, unless we are actually timing the students for a test, many teachers may say "In five minutes we will begin the next activity" and in fact, a time span greater than five minutes may pass before the actual transition in activities occur.

One major obstacle to the teaching of time is the previous knowledge of the base ten system which has ten digits to show all numbers. Time is more complex, having a base of 60 for minutes and seconds, as well as 12 hour clocks with a.m. and p.m. There is confusion, even for adults in understanding the difference between 12 a.m. and 12 p.m., in relation to noon and midnight. The time number line, with markings for easy numbered intervals like 1 minute, 5 minutes, 15 minutes, help the students make the transition when solving problems with "difficult time" like 7:43 p.m.

Objectives

Section 1 of the INVOCAB Curriculum Support Manual – Math Anxiety addresses the issue of math anxiety which may affect both teachers and students in the classroom environment and in real life situations. It begins with the identification of the symptoms of math anxiety and explores factors which may have contributed to its development. Critical to recognizing the need to eradicate math anxiety is the need to understand its effects on both the teacher and student. Practical strategies are shared about eliminating math anxiety and reducing its impact in the classroom. Both an informal measure of math anxiety and a formal scientific survey have been included in this manual with the aim of helping one to reflect on the extent it may exist in an individual.

Part of the new Primary School Curriculum, Curriculum Guides - Mathematics, includes the teaching of Poly's steps in problem solving and lists particular strategies to be taught in order to develop the necessary skills in solving problems effectively.

Section 2 of the INVOCAB Curriculum Support Manual – Problem Solving, examines common misconceptions about the aforementioned in the classroom. Polya's Steps in Problem Solving have been included to assist the teacher in being able to teach students how to approach problems to be solved in a strategic and systematic manner. A variety of problem solving strategies are included with worked examples of how the various strategies have been applied in finding the correct solution.

Section 3 of the INVOCAB Curriculum Support Manual - Recreational Mathematics, focuses on the three games or puzzles that can be used in the classroom to address several educational outcomes in Number, Problem Solving, Geometry and Measurement in the Mathematics Curriculum Guide for the Standard 3 – Standard 5 classes. In this section, an exception was made where outcomes from Measurement (Standard 2) were also included as a particular game shared was most suitable to develop those outcomes. In addition to the games and puzzles providing educational benefit, they help to promote positive attitudes towards mathematics and builds enthusiasm and persistence through solving the puzzles and games.

Section 4 of the INVOCAB Curriculum Support Manual – Multiplication Tables and Strategies, examines several strategies which provide opportunities for students to explore number patterns formed with repeated addition. Some employ colouring, the drawing of lines which can be adapted for string art, the use of counting with the fingers and tic tac toe grids to name a few. It is recommended to give the students the opportunity to explore the various multiplication strategies and make decisions on which one they appreciate and would use more frequently.

Section 5 of the INVOCAB Curriculum Support Manual – Elapsed Time – Strategies and Resources, shares various strategies for calculating elapsed time. The time number line is shown as a useful device to record and calculate elapsed time. Methods of recording elapsed time using various symbols suitable for different time spans are also shared. Also used is the strategy of Making A Table for problem solving and the use of a T-chart for calculating elapsed time, when given both the start and end times or given the elapsed time and either start time or end time and the students are required to calculate the other unknown time. Applying the "make a drawing" strategy for problem solving, the "Z-Method" has been included in the elapsed time section.

Perspectives on Instruction

The following offers a brief examination of various studies on how students learn. This would guide a teacher of mathematics on the implications in the classroom environment, strategies used and resources employed. The following paragraphs offer a few suggestions about what a teacher can do to improve instruction in the classroom so as to maximize student learning.

Jean Piaget (1977) has identified the third stage of cognitive development, where children between the ages of 7 -11, experience an accelerated development of their language and acquisition of basic skills. At this stage, classification is used as a logical operation to group objects according to criteria. Using the senses, students can examine 2 to 3 dimensions simultaneously. For example, children can explore solids and determine which solids can both stack and roll from those that can only roll. The implications of this is that teachers should not be fearful of challenging students to explore the manipulatives with given criteria. Do not underestimate the curiosity and interest shown by students, capitalise on it and provide more opportunities to explore.

Burns & Silbey (2000) argued that "hands-on experiences and multiple ways of representing a mathematical solution can be ways of fostering the development of this cognitive stage" (p. 55). Activities involving the use of manipulatives provide students the opportunity to understand abstract mathematical ideas and helps develop necessary skills for problem solving. For example, the use of concrete manipulatives like counters will help students build the multiplication tables with repeated addition. The use of the 0-99 chart will also assist with improving the understanding of skip counting, this in turn, will help develop the long term understanding and recollection of the tables, which will facilitate the application of the specific tables when solving word problems. When students can make their own time number line with markings of particular durations (whether 1 minute, 5 minute, 15 minute, 1 hour), it facilitates a deeper understanding of time, measures of time and the duration of time, which can all pose problems due to its abstract nature.

At this stage of development, children do not automatically make connections with concrete manipulations, abstract mathematics or symbolic representations (Burns & Silbey, 2000). When both modes are used in the classroom, teachers need to provide the opportunity to make linkages between them. This facilitative role is essential if students are to create meaning for themselves.

The classroom teacher has an active role to play in the teaching of mathematics, especially through demonstrating various strategies as well as the use of resources to the students. The teacher is most active at the stage of whole class instruction where strategies such as modelling and think alouds are used to engage students in reflection on what method was used, what was the reasoning behind using it, what procedures were followed etc. Over a period of time, there has to be a gradual shift away from the teacher's demonstration to the learner's independent application or working in small groups (Pearson & Gallagher, 1983). The teacher therefore must have a sound knowledge and understanding of the concepts, skills and strategies before modelling to the students, without a strong command of the content the presentation of material will lack accuracy and confidence. The timeframe between the teacher's modelling and the student's application should be gradual and not instantaneous. Teachers must have a realistic expectations of the students and must actively work towards the students becoming independent thinkers and problem solvers.

Peer tutoring was proven to have a profound effect on improving mathematical performance of students at risk and those facing difficulties in the learning of mathematics and computational skills at the elementary level (Kunsch, Jitendra & Sood, 2007). Teachers should explore the use of peer tutoring in the primary classrooms, where students with a stronger grasp of mathematical concepts are paired with weaker students, to share, explain and demonstrate what was done and why.

Problem Area 1: Math Anxiety

Republic Of Trinidad And Tobago, Ministry Of Education, Primary School Curriculum, Curriculum Guides, Mathematics, Infant 1 – Standard 5, Curriculum Planning and Development Division, 2013.

Excerpt from the Rationale – How is Mathematics structured?

Through an integrated approach, the new Primary Mathematics Curriculum aims to reduce "Math Anxiety" and Primary to Secondary transition issues by:

- *The development of core mathematical concepts and skills by the restructuring of learning activities to enable students to see connections with other subjects and their daily lives.*
- *The development of appropriate dispositions that would facilitate life-long learning and higher order thinking skills.*
- *A pedagogical approach that uses a variety of student-centred teaching techniques and strategies, such that improvement in student motivation and performance will increase in the medium and long terms.*

MATH ANXIETY

Have you ever witnessed, in a group of peers, someone being put on the spot to solve a complicated mathematical problem in a short space of time? The adult is expected to give the correct answer without hesitation.

Symptoms of Math Anxiety

In the situation presented above, experiencing any of the following symptoms is indicative of math anxiety: a queasy stomach, nausea, onset of sweaty hands and feet, an increased heart rate, clenched hands, hunched shoulders.

If the application of mathematical concepts or procedures in daily life brings feelings of diminished self-confidence, a sense of shame and isolation and a strong inability to cope, then it may be said that one is experiencing math anxiety.

If a teacher notices a strong urge to avoid teaching math or recognizes that there is certain discomfort with teaching specific math topics, this too is symptomatic of math anxiety.

At the end of this section, there are two assessments which have been used to measure levels of math anxiety. The first is an informal, non-scientific method which uses language to help a person

express feelings on Mathematics in a non-threatening way. The second assessment is a scientific survey designed by Rachel McAnallen, which has been used in a study on Math Anxiety.

Definition of Math Anxiety.

Richardson and Suinn (1972) defined mathematics anxiety as "feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations."

Factors which may have caused Math Anxiety

Various negative experiences in the past would have contributed to the development of Math Anxiety within a person. These include: negative peer interactions where classmates would tease someone for one's inability to perform simple math calculations; negative teacher interactions, where the teacher's attitude displayed impatience or hostility at a student's lack of comprehension with given methods resulting in the teacher scolding or making fun of the person's attempts to solve problems. The teacher may have even gone so far as to act in a threatening manner or become abusive and embarrass the student. Additional factors such as the classroom environment could also have contributed to a person's anxiety towards math. Examples of negative classroom environments would be ones in which emphasis is placed on competition or ranking students in terms of ability as well as those that place excessive pressure on students to solve math problems within a strict time frame.

Many teachers, upon reflection on various experiences of math learning, realize that the delivery of content was in the form of rote learning, which resulted in difficulty in transferring the understanding to specific teaching methods. Mathematical understanding may be based on the linear manner in which persons were taught. Gaps in understanding often occur when students are absent from school and teachers fail to revisit content. Additionally, in some cases, students lack the confidence to ask for clarification of confusing topics. This is all further compounded by the teacher's attitude towards the subject, especially if mathematics is not valued as being crucial to everyday life and not just as an academic endeavor.

Implications of Having Math Anxiety

The unusual thing about math anxiety is that it is highly transferable to others. Whatever impacted and shaped one's attitude toward math would form the basis of how it is taught, whether in the home or at school. Recognizing one's level of math anxiety is important, firstly to determine

whether or not it may have been unconsciously transferred to those in your charge and secondly, to halt and remedy its negative effects.

Steps to take to reduce your levels of Math Anxiety

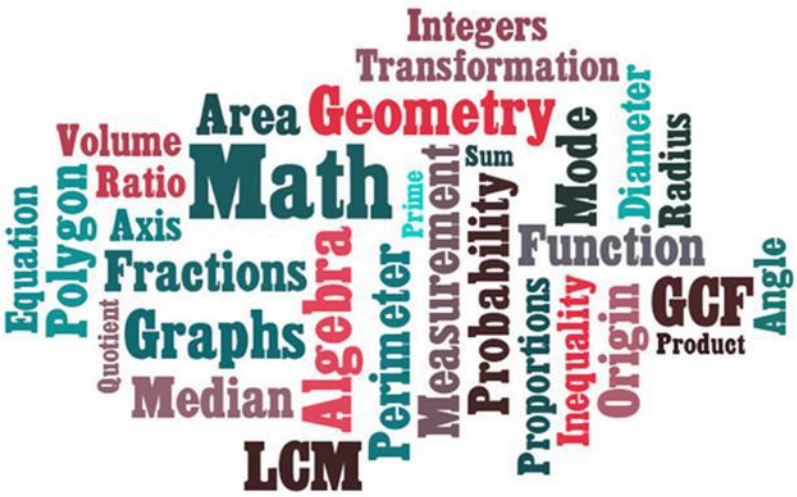
- Improve math understanding and confidence by pursuing additional content courses
- Examine past papers of various levels (SEA and CSEC) to review content and reflect on the methodology used for teaching
- Explore new mathematical manipulatives privately, in a relaxed frame of mind. (It is not advised to explore manipulatives in the classroom setting if its content and use have not been mastered)
- Understand that math anxiety affects both teachers and students. Develop a support network with other teachers/adults who have similar challenges. Share successful coping strategies.

Reducing Math Anxiety in the Classroom

- Make connections between math and daily life
- Point out the math in children's literature and in other subjects such as art and science
- Promote math vocabulary in everyday speech
- Ensure that learners are able to explore concepts and share ideas in a non-threatening classroom environment
- Provide opportunities for students to develop confidence through genuine problem solving
- Utilize role play as a problem solving strategy. Allow learners to express and rationalize what was done orally and through various forms of written communication (words, drawings and symbols)
- Use word problems that depict real life situations that are familiar to students
- Provide ample opportunities for individual and group work
- Be enthusiastic about the content, celebrate students' achievements no matter how small

IF MATH WERE A COLOUR?

Write a paragraph, explaining what mathematics would look like if it were a colour. Explain why you perceive mathematics as that colour as opposed to another colour.



McAnallen Anxiety in Mathematics Teaching Survey (MAMTS)

Que. 1 -25 - Please circle the number that best describes your level of agreement with the statement.

		Strongly Disagree	Disagree	Neither Agree Nor Disagree	Agree	Strongly Agree
1.	**I was one of the best math students when I was in school.**	1	2	3	4	5
2.	**Having to work with fractions causes me discomfort.**	1	2	3	4	5
3.	**I feel confident in my ability to teach mathematics to students in the grade I currently teach.**	1	2	3	4	5
4.	**I am confident that I can learn advanced math concepts.**	1	2	3	4	5
5.	**When teaching mathematics, I welcome students' questions.**	1	2	3	4	5
6.	**I have trouble finding alternative methods for teaching a math concept when a student is confused.**	1	2	3	4	5
7.	**I can easily do math calculations in my head.**	1	2	3	4	5
8.	**I find it difficult teach mathematical concepts to students.**	1	2	3	4	5
9.	**I feel confident using multiple resources when I teach.**	1	2	3	4	5
10.	**I don't have the math background to differentiate instruction for the most talented students in my class.**	1	2	3	4	5
11.	**I dislike having to teach math every day.**	1	2	3	4	5

		Strongly Disagree	Disagree	Neither Agree Nor Disagree	Agree	Strongly Agree
12.	**I avoid taking non-required math coursed in college.**	**1**	**2**	**3**	**4**	**5**
13.	**I am confident in my math abilities.**	**1**	**2**	**3**	**4**	**5**
14.	**I am confident that I can solve math problems on my own.**	**1**	**2**	**3**	**4**	**5**
15.	**I become anxious when I have to compute percentages.**	**1**	**2**	**3**	**4**	**5**
16.	**I have math anxiety.**	**1**	**2**	**3**	**4**	**5**
17.	**It makes me nervous to think about having to do any math problem.**	**1**	**2**	**3**	**4**	**5**
18.	**On average, other teachers are probably much more capable of teaching math that I am.**	**1**	**2**	**3**	**4**	**5**
19.	**I cringe when a student asks me a math question that I can't answer.**	**1**	**2**	**3**	**4**	**5**
20.	**I am comfortable working on a problem that involves algebra.**	**1**	**2**	**3**	**4**	**5**
21.	**I have a strong aptitude when it comes to math.**	**1**	**2**	**3**	**4**	**5**
22.	**I doubt that I will be able to improve my math teaching ability.**	**1**	**2**	**3**	**4**	**5**
23.	**If I don't know the answer to a student's math question, I have the ability to find the answer.**	**1**	**2**	**3**	**4**	**5**
24.	**I become anxious when a student finds a way to solve a problem with which I am not familiar.**	**1**	**2**	**3**	**4**	**5**

	Strongly Disagree	Disagree	Neither Agree Nor Disagree	Agree	Strongly Agree
25. **I would welcome the chance to have my supervisor evaluate my math teaching.**	1	2	3	4	5

26. I am a:
    - ☐ Male
    - ☐ Female

27. Number of Years of Mathematics Teaching Experience_____
    Current Grade Level you are teaching _____
    Highest Grade Level Taught_____

28. Place a check mark in front of the following math classes you successfully completed in high school:
    _____ Algebra I _____ Geometry _____Algebra II
    _____ Trigonometry/Precalculus _____ Calculus I _____ Calculus II

29. What is the highest level math class that you passed in college? _____

30. Compare yourself to other elementary teachers in terms of your mathematical abilities:
    - ☐ 1 One of the worst
    - ☐ 2 Way below average
    - ☐ 3 Below average
    - ☐ 4 Average
    - ☐ 5 Above average
    - ☐ 6 Way above average
    - ☐ 7 One of the best

31. Do you enjoy doing math?
    - ☐ Yes
    - ☐ No – Skip to Question 34

32. When did you first realize that you enjoyed mathematics?

- ☐ Primary school (K-2)
- ☐ Elementary (3-5)
- ☐ Middle school (6-8)
- ☐ High School (9-12)
- ☐ College / Adulthood
- ☐ Don‘t remember

33. Describe what you enjoy about mathematics:

_____

_____

_____

_____

_____

_____

_____

_____

34. Do you experience "math anxiety?"

- ☐ Yes
- ☐ No – Please continue to end.

35. Rate the degree of your math anxiety.

- ☐ 1 – Mild
- ☐ 2 - Moderate
- ☐ 3 – Severe

36. When did you first experience math anxiety?

- ☐ Primary school (K-2)
- ☐ Elementary (3-5)
- ☐ Middle school (6-8)
- ☐ High School (9-12)
- ☐ College / Adulthood
- ☐ Don't remember

37. Please describe the circumstances that led to your first experience with math anxiety (use the back of the paper if you need more space).

_____

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_____

_____

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Thank you for completing this survey and please return it to Rachel McAnallen.

Resources

Worksheet on If Math were a colour?

Reed, Kara Lianna. "Do K, 1, 2 teachers who participated in a yearlong math course have less teacher math anxiety than those who did not participate?" (2014). - McAnallen Anxiety in Mathematics Teaching Survey (MAMTS)

Further Reading

http://www.science.gov/topicpages/m/mathematics+anxiety+scale.html Science.gov Your Gateway to US Federal Science – Sample Records for Math Anxiety Scale

http://www.scholastic.com/teachers/article/conquering-math-anxiety Conquering Math Anxiety - Five strategies to help students who fear math. - Leah Shaffer

http://stemwire.org/2013/05/31/math-anxiety-doesnt-just-affect-students/ Math anxiety doesn't just affect students - Hetali Lodaya

http://www.slideshare.net/sme6044/mathematics-anxiety-12095318 SME 6044 - Contemporary Issues in Mathematics Education

http://rationalmathed.blogspot.com/2007/12/if-math-were-color-and-other-crimes.html - Dec 21 - "If math were a color. . ." and other crimes against humanity.

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Problem Area 2: Problem Solving

Republic Of Trinidad And Tobago, Ministry Of Education, Primary School Curriculum, Curriculum Guides, Mathematics, Infant 1 – Standard 5, Curriculum Planning and Development Division, 2013.

Standard 3

Whole Number (Operations): Addition and Subtraction

CONTENT

1.1.8. Solve real-life problems (concrete, pictorial and symbolic modes, including money) involving addition and subtraction.

1.1.9. Develop estimation skills.

1.1.10. Demonstrate an understanding of the relationship between addition and subtraction.

SKILLS

1.2.15. Solve problems involving addition (up to 4 digit numbers with sum less than 10 000) and up to 4 addends and subtraction (with minuend up to 4 digits).

1.2.16. Determine the reasonableness of answers by using estimation.

1.2.17. Use the inverse operation to check answers.

1.2.18. Explain or demonstrate how an answer was obtained when solving problems.

DISPOSITIONS

1.3.5. Appreciate the use of algorithms in solving problems involving the operations (addition and subtraction).

OUTCOMES

11. Demonstrate an understanding of the algorithm for addition and subtraction.
12. Solve a variety of word problems using problem solving strategies including mental strategies.
13. Demonstrate an understanding of estimation skills.
14. Use the relationship between addition and subtraction to check answers.

ELABORATIONS

- Solve one-step and multi-step addition and subtraction problems involving whole numbers and money (including bills, best buy, profit and loss, using dollars only and cents only) by:
    - Using a variety of problem solving strategies, such as: use a model, act it out, draw a picture, look for a pattern, guess and check, work backwards, logical reasoning, make a table or chart, make an organized list and,/or try a simpler form of the problem.
    - Using the algorithm.
    - Using mathematical games.
    - Creating number sentences with one unknown.
    - Using estimation skills to check solutions to problems.
    - Using the reverse operation to check answers.
    - Recording solutions to problems using drawings, numerals, symbols and words. [1.1.8, 1.1.9, 1.1.10, 1.2.15, 1.2.16, 1.2.17, 1.3.5].
- Explain or demonstrate how an answer was obtained when solving problems. [1.1.8, 1.2.18, 1.3.5].

Standard 3

Whole Number (Operations): Multiplication and Division.

CONTENT

1.1.13. Solve real-life problems (concrete, pictorial and symbolic modes, including money) involving multiplication and division.

1.1.14. Develop estimation skills.

1.1.15. Demonstrate an understanding of the relationship between multiplication and division.

SKILLS

1.2.21. Solve real-life problems involving multiplication (up to 2 digit by 2 digit numbers) and division (up to 4 digit divided by 1 digit).

1.2.24. Explain or demonstrate how an answer was obtained when solving problems.

DISPOSITIONS

1.3.6. Appreciate the use of algorithms in solving problems involving the operations (multiplication and division).

OUTCOMES

16. Develop and apply procedures to multiply and divide whole numbers to solve problems.
17. Solve a variety of word problems using problem solving strategies including mental strategies.
18. Demonstrate an understanding of estimation skill.
19. Use the relationship between multiplication and division to check answers.

ELABORATIONS

- Solve one-step and multi-step multiplication and division problems (including problems involving the unitary method), involving whole numbers and money (including bills, best buy, profit and loss, rate (weekly, hourly, daily, monthly, yearly and by the minute - using dollars only and cents only) by:
    - Using a variety of problem solving strategies, such as: use a model, act it out, draw a picture, look for a pattern, guess and check, work backwards, logical reasoning, make a table or chart, make an organized list and try a simpler form of the problem.

Standard 4

PROBLEM SOLVING

CONTENT

1.1.21. Solve multi-step problems involving whole numbers, fractions and decimals using a variety of strategies.

SKILLS

1.2.30. Solve one-step and multi-step problems involving whole numbers, fractions and decimals (including money transactions, bills, best buy, profit and loss) using the four operations and a variety of strategies.

DISPOSITIONS

1.3.8. Develop confidence in working independently in selecting and using various mental and written strategies to solve problems.

OUTCOMES

16. Solve multistep problems involving whole numbers, fractions and decimals using algorithms, mental strategies and other problem solving strategies.
17. Solve problems involving direct proportion.

ELABORATIONS

- Solve routine and non-routine problems using a variety of strategies such as: use a model, act it out, draw a picture, look for a pattern, guess and check, work backwards, logical reasoning, make a table or chart, make an organized list and try a simpler form of the problem. [1.1.21, 1.2.30, 1.3.8].

Standard 5

PROBLEM SOLVING

CONTENT

1.1.8. Create and solve single and multistep problems involving the four operations.

1.1.9. Apply mental mathematics strategies to solve problems.

1.1.10. Use patterns and other strategies to solve problems.

SKILLS

1.2.15. Create and solve real-life, one-step and multi-step problems involving operations with whole numbers, fractions, mixed numbers, decimals, simple percents and money, (including profit and loss, discount, savings, salaries, wages, loans, simple interest and VAT).

1.2.16. Solve problems involving unequal sharing (not including the use of ratio).

DISPOSITIONS

1.3.5. Demonstrate appropriate judgment in selecting problem solving strategies.

1.3.6. Develop flexibility in using a variety of strategies to solve problems.

1.3.7. Demonstrate independence and perseverance in solving problems.

OUTCOMES

8. Create and solve one-step and multi-step problems involving whole numbers, fractions, mixed numbers, decimals, percents, including money, using algorithms, mental strategies and other problem solving strategies.
9. Solve problems involving unequal sharing.

ELABORATIONS

- Solve routine and non-routine problems using a variety of strategies such as: use a model, act it out, draw a picture, look for a pattern, guess and check, work backwards, logical reasoning, make a table or chart, make an organized list, and try a simpler form of the problem. [1.1.10, 1.2.15, 1.3.5, 1.3.6, 1.3.7]

Suggestions on the teaching of Problem Solving

1. Vary the problems at the end of a lesson or unit, where different operations are involved. This reduces the possibility of the learner believing that the only strategy for solving all of the problems presented is the strategy just presented in the lesson. The variety of problems to be solved provides the students with the opportunity to carefully examine each problem in order to determine which operation is required to solve the problem.
2. Exposure to multi-step problems based on real life experiences and situations will be beneficial to students as they help promote logical and abstract thinking; skills which will help prepare them for future learning.
3. Actively teach students how to problem solve, this includes teaching the process of problem solving using Polya's Steps in addition to the various strategies used for solving a problem. In this way, students will be able to properly develop concepts, content and algorithms. In subsequent lessons, consider using problems as a springboard to teach various concepts, as the correct problem, when presented, can trigger deep analysis.
4. Ensure that a variety of problems are offered, which would give the students the opportunity to solve using different strategies. Encourage students to share the strategies they have applied and explain the reasoning behind their use. Allow students to examine the strategies applied by others and compare them with their own choices, based on efficiency and practicality. Promote the understanding that there may be more than one way to solve a problem.

Suggested Teaching Strategies

APPLYING POLYA'S FOUR STEPS

Step 1 -Understanding the Problem

- What is the problem about?
- What data is known?
- What is the question asking?

  Allow students the opportunity to verbally state the problem in their own words, explaining what information they were given and what is being asked of them to find out. Some students may need to underline or highlight critical data from the problem to increase their understanding.

Step 2 - Devising a Plan

- How will you solve the problem?
- What is the best strategy? - *Students brainstorm to select the possible strategies which may be applicable, then select the best strategy for solving this problem.*

Step 3 - Carry out The Plan

- Implement the strategy. - *Students apply the chosen strategy*

Step 4 - Look Back

- Did the chosen solution, answer the problem from step 1?
- Check the answer using another strategy.
- Review step 3 to see if any mistakes were made - e*nsure that students review the steps followed to ensure no errors were made*
- Go back to step 2 and choose another strategy - *encourage the students to double check the solution by using another strategy to solve the problem.*

(How to Solve It, Polya, 1945).

VARIOUS STRATEGIES USED TO SOLVE GIVEN PROBLEMS

Problem – Sharing Pencils

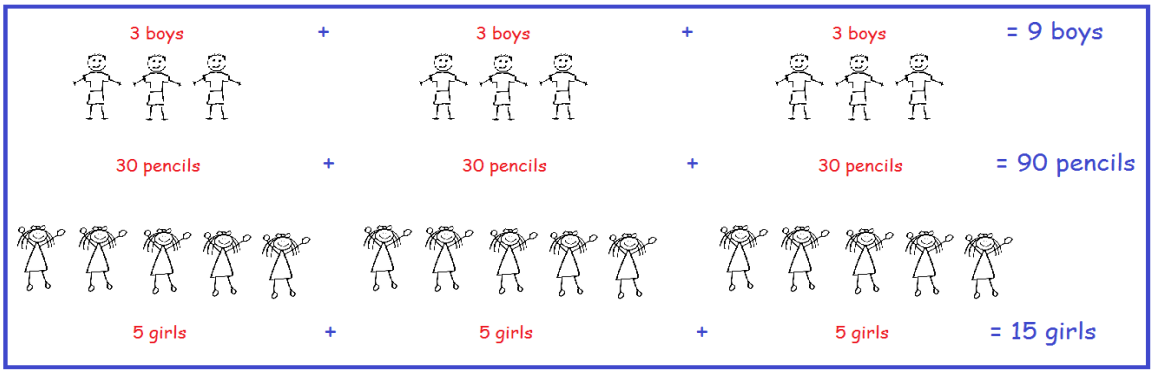
A teacher shares pencils among his students. Half of the pencils were shared equally among 9 boys. The other half of the pencils were shared equally among 15 girls.

a) If 3 boys and 5 girls received 30 pencils, what was the total number of pencils shared?

b) How many pencils did each girl receive?

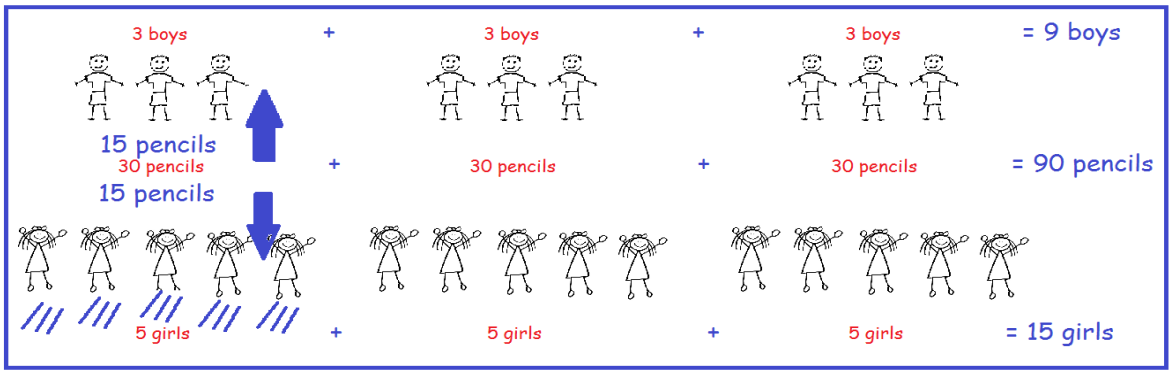
STRATEGY 1 - DRAW A PICTURE

i. Draw one group of 3 boys and 5 girls. Write the number of pencils the group/all children received (30 pencils). Repeat the drawing until all 9 boys and 15 girls are represented, with each group receiving 30 pencils. Tally the total number of pencils (30 +30+30 = 90).



ii. Choosing one group, let the drawing reflect half of the 30 pencils for the boys and the other half, for the girls. Draw the 15 pencils as they are shared among the 5 girls. (15 ÷5= 3).

a) If 3 boys and 5 girls received 30 pencils, what was the total number of pencils shared?



Answer – 90 pencils.

b) How many pencils did each girl receive? ***Answer – 3 pencils.***

Problem – Sharing Pencils (see problem above)

STRATEGY 2 - ACT IT OUT

i. Enlist students to enact the problem. Role play with 9 boys and 15 girls. Separate the 9 boys and 15 girls into groups with 3 boys and 5 girls each (forming 3 groups). Use 30 pencil props for each group (making a sum of 90 pencils).

ii. Split the 30 pencils within each group in half. Give half (15) to the boys and share the remaining 15 pencils among the 5 girls. Each girl gets 3 pencils.

a) If 3 boys and 5 girls received 30 pencils, what was the total number of pencils shared? ***Answer – 90 pencils.***

b) How many pencils did each girl receive? ***Answer – 3 pencils.***

Problem – Brownies.

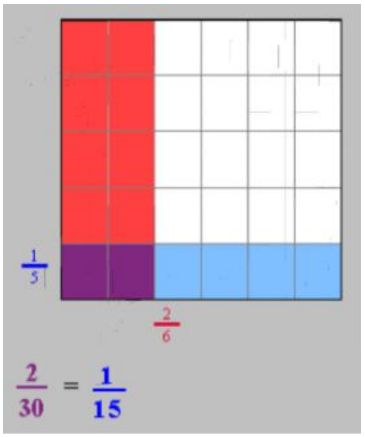
2/6 of a pan of brownies was on the counter. Chen packed away 1/5 of the brownies in the pan. How much of the whole pan of brownies was packed away?

STRATEGY 3 - USE A MODEL

i. Use an area model to represent the pan of brownies. A rectangular grid, 6 squares long and 5 squares wide represents the entire pan.

ii. In the drawing shown on the right :-

- the red colour reflects the 2/6 of the pan of brownies,
- the blue colour reflects the 1/5 of the pan,
- the intersection of red and blue colours - the purple reflects the fraction of the whole pan of brownies packed away.



How much of the whole pan of brownies was packed away?

Answer – 1/15.

Problem – Building a Pyramid with blocks.

Johnny builds a pyramid with blocks. He put 1 block in the top row. 3 blocks in the second row, and 5 blocks in the third row. If he continues this pattern, how many blocks would Johnny put in the 7th row?

STRATEGY 4 - LOOK FOR A PATTERN

i. A table is made where the relevant information is included.

Row	*1*	*2*	*3*	*4*	*5*	*6*	*7*
Blocks	*1*	*3*	*5*				

ii. A pattern is noted where 2 is added to the number of blocks in the previous row, starting from the 1 block in the 1st row. This will give the number of blocks in the subsequent rows. Using this pattern, the students can calculate the number of blocks in the 7th row.

Row	*1*	*2*	*3*	*4*	*5*	*6*	*7*
Blocks	1^{+2}	3^{+2}	5^{+2}	7^{+2}	9^{+2}	11^{+2}	*13*

How many blocks did Johnny put in the 7th row? **Answer – 13 blocks.**

Problem – Field Trip.

On a field trip, a fee of $40.00 is paid for an adult and $25.00 for a child. A total sum of $755.00 is collected from 20 people. How many children were on this trip?

STRATEGY 5 - GUESS AND CHECK

i. Make a table to record guesses. Start with a reasonable number of 10 adults and 10 children which make up the total number of people (20). Find the total sum paid.

ii. Experiment with different combinations of adults and children, with a total of 20 people. Find the sum for the combinations used.

No. of Adults	**Amt.** *$40*	**No. of Children**	**Amt.** *$25*	**Total Sum**	
10	$400	10	$250	$650	Total is not enough, add more adults, put less children. Make sure to make 20.
12	$480	8	$200	$680	Total is still not enough, add more adults, use less children
15	$600	5	$125	$725	Total is still not enough, add more adults, use less children
17	$680	3	$ 75	$755	**Correct Total.**

How many children were on this trip? **Answer – 3 children.**

Problem – Gnomes and the Magic Forest

(retrieved from http://mps.k12.vt.us/msms/math/gnomes.html)

The community of gnomes in the Magic Forest has to move as their forest is being cut down. They are leaving in boats. Each boat can hold up to and including 100 grams and stay afloat. The gnomes come in five different weights: 60 grams (senior citizens), 40 grams (adults), 20 grams (teenagers), 10 grams (children), and 5 grams (babies). What are the possible combinations of gnomes that can be put safely into a boat?

STRATEGY 6 - MAKE A TABLE

SENIOR CITIZENS	ADULTS	TEENAGERS	CHILDREN	BABIES	TOTAL
60 g	40 g	20 g	10 g	5 g	
1	1	0	0	0	100
1	0	2	0	0	100
1	0	1	2	0	100
1	0	1	1	2	100
1	0	1	0	4	100
1	0	0	4	0	100
1	0	0	3	2	100
1	0	0	2	4	100
1	0	0	1	6	100
1	0	0	0	8	100

Problem – Bus Stop.

There were some passengers on a bus as it left Bus Stop A. At Bus Stop B, 15 passengers got off and 20 got on. At Bus Stop C, 9 passengers got on and none got off. Then there were 45 passengers on board. How many passengers were on the bus at Bus Stop A?

STRATEGY 7 - WORK BACKWARDS

45 passengers on board. 9 got on. So I take 9 from 45. (45 – 9 = 36),

20 got on, so I take 20 from 36. (36 – 20 = 16),

15 got off, so I add 15 to 16. (16 + 15 = 31).

How many passengers were on the bus at bus stop A? **Answer 31 passengers**.

Problem- Telecommunication Services.

A telecommunications solutions provider offers various services for customers to choose from. Customers are allowed to choose either 1 service or 2 services. A total of 43 customers selected various services. 21 customers selected the landline service and 28 customers selected the internet service. How many customers selected both services?

STRATEGY 8 - LOGICAL REASONING

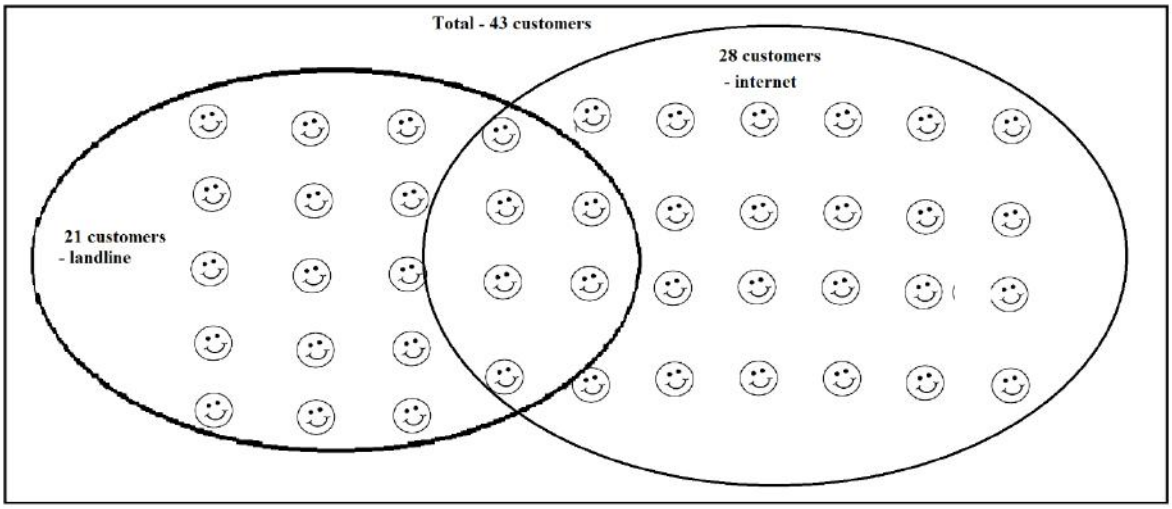
i. Draw the 43 customers.

ii. Identify the 21customers who selected the landline service by making a loop around 21 of the customers.

iii. Identify the 28 customers who selected the internet service by making a loop around 28 customers. (This is difficult as there are less than 28 customers.) Extend the loop for the 28 customers by including enough customers from the landline customers to reflect the entire 28 customers.

iv. Count the number of customers that are within the area where there is an overlap of both loops.

How many customers selected two services? **Answer – 6 customers**



Resources

http://nlvm.usu.edu/en/nav/vlibrary.html National Library of Virtual Manipulatives, Utah State University.

Further Reading

www.math.utah.edu/~pa/math/**polya**.html Summary taken from G. Polya, "How to Solve It", 2nd ed., Princeton University Press, 1957, ISBN 0-691-08097-6. UNDERSTANDING THE PROBLEM. First.

http://teacher.scholastic.com/lessonrepro/lessonplans/steppro.htm Four Steps to Problem Solving

http://www.mathstories.com/strategies.htm Math Stories. Com Math for Internet Generation, Problem Solving Strategies.

https://nzmaths.co.nz/problem-solving-strategies NZMaths. Problem Solving Strategies

https://www.google.tt/url?sa=t&rct=j&q=&esrc=s&source=web&cd=3&ved=0ahUKEwikm4DHs73PAhUCax4KHcbTA8oQFggqMAI&url=http%3A%2F%2Ffloridarti.usf.edu%2Fresources%2Fformat%2Fpdf%2FClassroom%2520Cognitive%2520and%2520Metacognitive%2520Strategies%2520for%2520Teachers_Revised_SR_09.08.10.pdf&usg=AFQjCNFCgm1V4UmP1HXwNgWVPkb5j34FtA&sig2=3phrHf1bPXZ6nyEsHk0RqA&bvm=bv.134495766,d.dmo&cad=rjt
Classroom Cognitive and Metacognitive Strategies for Teachers Revised SR 09.08.10

Problem Area 3: Recreational Mathematics

Republic Of Trinidad And Tobago, Ministry Of Education, Primary School Curriculum, Curriculum Guides, Mathematics, Infant 1 – Standard 5, Curriculum Planning and Development Division, 2013.

Standard 4

Number - Problem Solving

CONTENT

1.1.21 Solve multi-step problems involving whole numbers…. using a variety of strategies.

SKILLS

1.2.30 Solve one-step and multi-step problems involving whole numbers…, using … a variety of strategies.

1.2.32 Investigate and apply mental mathematics strategies and skills to solve problems.

DISPOSITIONS

1.3.8 Develop confidence in working independently in selecting and using various mental and written strategies to solve problems.

OUTCOMES

16. Solve multi-step problems involving whole numbers, fractions and decimals using algorithms, mental strategies and other problem solving strategies.

ELABORATIONS

- Solve routine and non-routine problems using a variety of strategies such as: use a model, act it out, draw a picture, look for a pattern, guess and check, work backwards, logical reasoning, make a table or chart, make an organized list and try a simpler form of the problem. [1.1.21, 1.2.30, 1.3.8]
- Solve problems in mathematical games. [1.1.21, 1.2.30, 1.3.8].

Standard 5

Number - Problem Solving

CONTENT

1.1.10 Use patterns and other strategies to solve problems.

DISPOSITIONS

1.3.5 Demonstrate appropriate judgment in selecting problem solving strategies.
1.3.6 Develop flexibility in using a variety of strategies to solve problems.
1.3.7 Demonstrate independence and perseverance in solving problems.

OUTCOMES

8. Create and solve one-step and multi-step problems involving whole numbers.

ELABORATIONS

- Select and use appropriate mental strategies to aid in finding solutions to problems. [1.1.9, 1.2.15, 1.3.5, 1.3.6].

- Solve routine and non-routine problems using a variety of strategies such as: use a model, act it out, draw a picture, look for a pattern, guess and check, work backwards, logical reasoning, make a table or chart, make an organized list, and try a simpler form of the problem. [1.1.10, 1.2.15, 1.3.5, 1.3.6, 1.3.7]
- Solve problems in mathematical games. [1.1.10, 1.2.15, 1.3.7].

Standard 3

Geometry – Plane Shapes

CONTENT

2.1.4 Develop spatial sense through explorations in relation to plane shapes.

2.1.5 Investigate properties of plane shapes.

2.1.6 Solve problems involving plane shapes.

SKILLS

2.2.5 Compare and classify plane shapes according to their properties.

2.2.6 Differentiate between regular and irregular polygons (triangles, quadrilaterals, pentagons, hexagons, octagons).

2.2.7 Explore the effects of orientation change on plane shapes.

2.2.8 Solve problems involving plane shapes.

DISPOSITIONS

2.3.3 Display confidence in exploring plane shapes.

OUTCOMES

3. Demonstrate an understanding of the properties of plane shapes.

4. Solve problems involving plane shapes.

ELABORATIONS

- Examine and describe the properties of regular and irregular polygons. [2.1.4, 2.1.5, 2.2.5, 2.3.3].
- Solve problems involving plane shapes. [2.1.6, 2.2.8, 2.3.3].

Standard 4

Geometry - Solids and Plane Shapes

CONTENT

2.1.1 Develop an understanding of the properties of solids and plane shapes.

2.1.2 Solve problems involving solids and plane shapes.

SKILLS

2.2.2 Construct and draw regular and irregular polygons given their properties, using the principles of parallel and perpendicular lines, angles and number of sides.

2.2.3 Solve problems involving solids and plane shapes.

DISPOSITIONS

2.3.1 Display collaboration while working in groups.

OUTCOMES

1. Demonstrate an understanding of the properties of solids and plane shapes.
2. Solve problems involving solids and plane shapes.

ELABORATIONS

- Construct and draw plane shapes given a description of its properties and using appropriate resources including computer software. [2.1.1, 2.2.2, 2.3.1].
- Solve problems involving solids and plane shapes. [2.1.2, 2.2.3, 2.3.1].

Standard 5

Geometry - Solids and Plane Shapes

CONTENT

2.1.2 Develop spatial sense through exploration of solids and plane shapes.

2.1.4 Solve problems involving solids and plane shapes.

SKILLS

2.2.4 Solve problems involving solids and plane shapes.

DISPOSITIONS

2.3.1 Display curiosity while exploring solids and plane shapes.

2.3.2 Develop perseverance when solving problems involving shape and space.

OUTCOMES

3. Solve problems involving solids and plane shapes.

ELABORATIONS

- Solve problems involving plane shapes. [2.1.4, 2.2.4, 2.3.2].

Standard 3

Measurement - Area

CONTENT

3.1.17 Understand that measures can be quantified using standard units.

3.1.18 Develop measurement sense and apply appropriate techniques when measuring, making comparisons, approximating and estimating.

3.1.19 Demonstrate an understanding about the conservation of area.

3.1.20 Solve problems involving area.

SKILLS

3.2.25 Explain the need for and the importance of a standard unit of measure for area.

3.2.28 Calculate area of plane shapes on a grid with unit squares.

3.2.29 Explore the conservation of area.

3.2.30 Solve problems involving area.

DISPOSITIONS

3.3.9 Appreciate measures in everyday use.

3.3.10 Display a sense of inventiveness in selecting standard units when measuring area.

3.3.11 Demonstrate confidence in their abilities to calculate and compare area.

OUTCOMES

10. Demonstrate an understanding of measures of area.

11. Solve problems involving measures of area.

ELABORATIONS

- Explain the need for and the importance of a standard unit of measure for area. [3.1.17, 3.2.25, 3.3.9].
- Compare and order area of surfaces and explain reasoning, using appropriate vocabulary. [3.1.18, 3.2.27, 3.3.11].
- Select the appropriate unit of measure when measuring surfaces of varying sizes and explain the suitability of the unit. [3.1.18, 3.2.26, 3.3.10].
- Draw different shapes on grids that have the same area. [3.1.20, 3.2.28, 3.3.11].
- Explore activities associated with conservation of area and state findings/generalizations. [3.1.19, 3.2.29, 3.3.11].
- Solve problems involving area. [3.1.20, 3.2.30, 3.3.11].

Standard 4

Measurement – Area

CONTENT

3.1.18 Apply measurement techniques to quantify measures for area.

3.1.19 Solve problems involving area.

SKILLS

3.2.30 Calculate the area of compound shapes.

3.2.31 Draw shapes on a grid given the area of the shapes.

3.2.33 Solve problems involving area.

DISPOSITIONS

3.3.6 Develop confidence in their abilities to use concepts in area and to estimate and measure area.

OUTCOMES

12. Demonstrate an understanding of area of regular and irregular plane shapes.

ELABORATIONS

- Calculate the areas of compound shapes that may be dissected into rectangles and squares. [3.1.18, 3.2.30, 3.3.6].
- Draw different shapes of a given area on grids. [3.1.18, 3.2.31, 3.3.6].
- Solve problems involving area. [3.1.19, 3.2.33, 3.3.6].

Standard 5

Measurement – Area

CONTENT

3.1.8 Develop and apply formula for measurement of area.

3.1.9 Solve problems in real-life situations involving area.

SKILLS

3.2.8 Develop and use formula to calculate the area of squares and rectangles.

3.2.9 Solve problems involving area of compound shapes.

DISPOSITIONS

3.3.7 Appreciate the importance of formula for calculations involving area.

3.3.8 Display confidence when working independently in solving problems.

OUTCOMES

7. Develop and use proficiently, formula to calculate area in problem solving.

ELABORATIONS

- Generalize a rule (formula) for determining the area of squares and rectangles. [3.1.8, 3.2.8, 3.3.7].
- Write and explain the formula for finding the area of squares and rectangles. [3.1.8, 3.2.8, 3.3.7].
- Apply formula to find the areas of simple composite figures that may be dissected into rectangles and squares. [3.1.8, 3.1.9, 3.2.8, 3.2.9, 3.3.7, 3.3.8].
- Use estimation strategies to check for reasonableness of solutions to problems relating to area. [3.1.9, 3.2.8, 3.2.9, 3.3.8].
- Solve problems in real-life contexts involving area. [3.1.9, 3.2.8, 3.2.9, 3.3.7', 3.3.8].
- Solve problems involving perimeter and area. [3.1.9, 3.2.9, 3.3.8].

Standard 3 –

Measurement – Linear - Perimeter

CONTENT

3.1.4 Develop concept of perimeter using regular and irregular shapes.

3.1.5 Solve problems involving measures.

SKILLS

3.2.5 Solve computational problems and real-life problem involving length.

3.2.6 Differentiate between area and perimeter.

3.2.7 Measure and calculate the perimeter of regular and irregular shapes and compare and order.

3.2.8 Solve problems involving perimeter.

DISPOSITIONS

3.3.1 Appreciate the functional role of the linear measures in their everyday lives.

3.3.2 Demonstrate confidence in their abilities to estimate and measure, and in solving problems related to linear measure.

OUTCOMES

1. Apply measurement principles, including using an instrument, estimation and approximation to solve a wide variety of practical problems.
2. Develop a conceptual understanding of perimeter.
3. Solve problem involving linear measure.

ELABORATIONS

- Investigate the distance around shapes to determine the perimeter of shapes. [3.1.4, 3.2.6, 3.3.1].
- Explain the difference between area and perimeter. [3.1.4, 3.2.6, 3.3.1].
- Measure the perimeter of shapes using standard units. [3.1.4, 3.2.7, 3.3.1].
- Count and record the number of units used to measure the perimeter of a shape. [3.1.4, 3.2.7, 3.3.1].
- Compare and order the perimeter of two or more shapes and explain reasons using appropriate language. [3.1.4, 3.2.7, 3.3.1].
- Calculate the perimeter of regular and irregular plane shapes. [3.1.5, 3.2.8, 3.3.2].
- Solve problems involving perimeter. [3.1.5, 3.2.8, 3.3.2].

Standard 4 -

Measurement – Linear - Perimeter

CONTENT

3.1.1 Understand that measures can be quantified using standard units and their sub-parts.

3.1.2 Apply measurement techniques to quantify measures for length.

3.1.3 Solve problems involving linear measure.

SKILLS

3.2.6 Draw plane shapes given the perimeter.

3.2.7 Solve problems involving linear measure.

DISPOSITIONS

3.3.1 Appreciate the functional role of measurement in their everyday lives.

OUTCOMES

2. Demonstrate appropriate techniques when measuring.
3. Solve problems involving linear measure.

ELABORATIONS

- Construct or draw two or more rectangles for a given perimeter in a problem-solving context. [3.1.3, 3.2.6, 3.3.1].
- Measure and record the perimeter of a given irregular shape, and explain the strategy used. [3.1.1, 3.1.3, 3.2.6, 3.2.7, 3.3.1].
- Solve problems involving length. [3.1.1, 3.1.3, 3.2.7, 3.3.1].

Standard 5 -

Measurement – Linear - Perimeter

CONTENT

3.1.1 Develop and apply formulae for measurement of perimeter.

3.1.2 Solve problems in real life situations involving perimeter.

SKILLS

3.2.1 Develop and use formulae for finding the perimeter of squares and rectangles.

3.2.2 Solve problems involving perimeter of compound shapes.

DISPOSITIONS

3.3.1 Appreciate the functional role of measurement in their everyday lives.

3.3.2 Appreciate the importance of formulae for calculations in perimeter.

OUTCOMES

1. Develop and use proficiently, the formulae to calculate perimeter of squares and rectangles in problem solving.

ELABORATIONS

- Determine the perimeter of rectangles and squares. [3.1.1, 3.2.1, 3.3.1].
- Write and explain the formulae for finding the perimeter of any given rectangle and square. [3.1.1, 3.2.1, 3.3.1].
- Calculate and compare perimeters of squares and rectangles. [3.1.2, 3.2.1, 3.3.2].
- Find the perimeters of simple composite figures that may be dissected into rectangles and squares.

Standard 2 –

Measurement – Area

CONTENT

3.1.21 Develop the concept of area.

3.1.22 Demonstrate familiarity with comparison of objects using appropriate vocabulary.

3.1.23 Understand that measures can be quantified.

3.1.24 Apply measurement techniques to quantify measures.

3.1.25 Solve problems involving measures.

SKILLS

3.2.34 Touch, colour and cover surfaces to develop the concept of area.

3.2.36 Measure, record, compare and order area of surfaces using non-standard units.

3.2.37 Investigate which plane shapes are appropriate for measuring area.

3.2.38 Calculate the area of shapes by counting squares.

3.2.39 Solve problems involving area.

DISPOSITIONS

3.3.5 Demonstrate confidence in one's abilities to measure and compare.

OUTCOMES

21. Demonstrate an understanding of area.

22. Develop an understanding of the comparison of measures.

24. Demonstrate appropriate techniques when measuring.

25. Solve problem involving measures.

ELABORATIONS

- Investigate which plane shapes are appropriate for measuring area through tessellation activities and explain findings. [3.1.25, 3.2.37].
- Calculate the area of shapes by counting squares (of different sizes). [3.1.24, 3.1.25, 3.2.38, 3.3.5].
- Solve problems involving area. [3.1.25, 3.2.39].

Standard 3 -

Geometry – Symmetry

CONTENT

2.1.7 Develop an understanding of line symmetry.

2.1.8 Solve problems involving line symmetry.

SKILLS

2.2.9 Classify shapes into those that are symmetrical and those that are not.

2.2.10 Determine the number of lines of symmetry in plane shapes – regular, irregular and curved, and in numerals and letters.

2.2.11 Create symmetrical shapes.

2.2.12 Solve problems involving line symmetry.

DISPOSITIONS

2.3.4 Display curiosity while investigating lines of symmetry.

OUTCOMES

5. Demonstrate an understanding of the concept of line symmetry.

6. Solve problems involving line symmetry.

ELABORATIONS

- Determine whether or not plane shapes are symmetrical by folding and superimposing (and/or by using a Mira). [2.1.7, 2.2.9, 2.3.4].
- Investigate plane shapes, letters and numerals to determine whether or not they are symmetrical and to determine the number of lines of symmetry. [2.1.7, 2.2.10, 2.3.4].
- Create symmetrical shapes. [2.1.8, 2.2.11, 2.3.4].
- Complete a symmetrical shape, given half of the shape and a line of symmetry. [2.1.8, 2.2.11, 2.3.4].
- Solve problems involving line symmetry. [2.1.8, 2.2.12, 2.3.4].

Common Issues

The new Primary Mathematics Curriculum aims to reduce "Math anxiety" and ease the transition from Primary to Secondary school "through the development of appropriate dispositions that would facilitate life-long learning and higher order thinking skills" (Pg.5). Reference is made to "Adding It Up: Helping Children Learn Mathematics" (2001), where it is argued that in developing mathematical proficiency, instructional programs need to include methodology which develops strategic competence in formulating, representing and solving mathematical problems and adaptive reasoning on logical thought, reflection, explanation and justification.

Teachers may easily dismiss mathematical games, not realizing the benefits of using them in the classroom as the mathematical benefits are not overt, and the valuable time spent solving such puzzles may be perceived to be better spent teaching algorithms and problem solving skills.

Background Information

Recreational mathematics, presented in the form of puzzles or brain teasers, requires the participants to use the logical and deductive skills of mathematics to find solutions to problems. It encourages the participants to be reflective and take risks as they attempt to solve mathematical problems. Through recreational mathematics, participants gain new knowledge without being aware that they are learning mathematical content

Recreational Mathematics does not require the participants to possess superior math skills. As such there is no threat to the user in attempting to solve a puzzle, neither does it bring any shame or embarrassment if errors are made. When used by adults and young learners, it promotes persistency and curiosity as users have the opportunity to self-correct.

These fun activities can be presented in the form of tangible resources like wooden or plastic tangram puzzles, a page with a printed Sudoku puzzle, using ICT resources such as websites, interactive online math game applications (apps) and software.

Suggested Puzzles

SUDOKU PUZZLES

Sudoku puzzles are grounded in mathematical theory involving:

- combinatorics which is employed in the counting of the Sudoku grids
- group theory when examining when two grids are equivalent
- computational complexity in the solving of the Sudokus

Some Sudoku puzzles use the numbers 1 -9. *Instead of digits, other symbols can be used, e.g. letters, as long as there are nine different symbols.* Many people mistakenly think that arithmetic skills are needed to solve the puzzles. While no operations are applied in solving these problems involving numbers, the process of solving requires other mathematical skills such as logical deduction, elimination, the application of trial and error and back tracking.

To solve a Sudoku puzzle, one needs to use a combination of logic and trial-and-error. More math is involved behind the scenes: combinatorics (used in counting valid Sudoku grids), group theory (used to describe ideas of when two grids are equivalent) and computational complexity (with regards to solving the Sudoku puzzle).

SUDOKU PUZZLE

	5	9		2		4	6	
1	2		4		3	7		8
3		4		7		9		2
	3		8	5	9		2	
6		5		4		3		7
	1		7		6		4	9
2		3		1	7		9	4
9			3		2	6		5
	7	8	9	6		2	3	

In Japanese, Sudoku means *"numbers singly"*.

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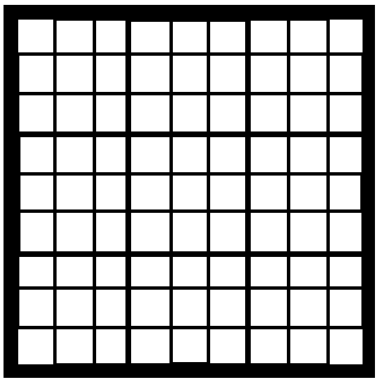
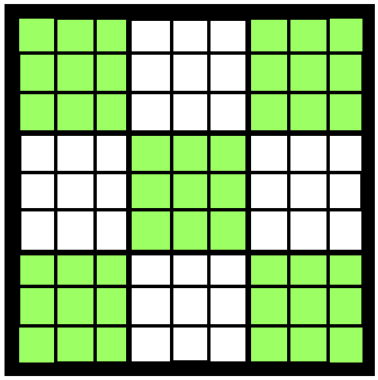
- A Sudoku puzzle is a grid of nine by nine s**quares** or **cells.**



9 x 9 squares or cells



9 subgrids or regions of 3x3 cells

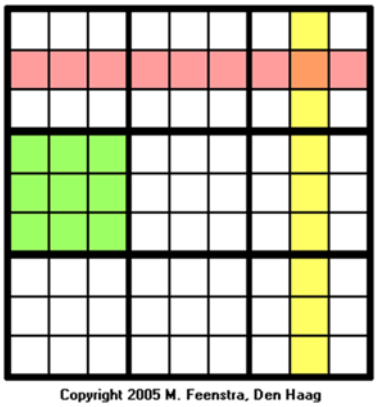
- The grid is sub-divided into nine **sub-grids** or "**regions**" of three by three cells.

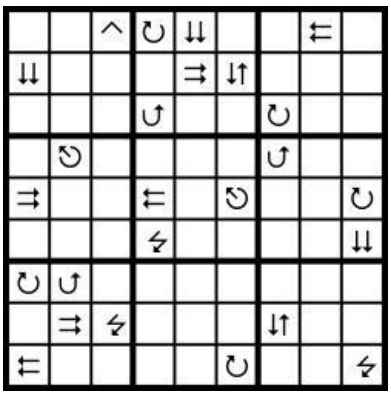
- *The objective of Sudoku* is to enter a digit from 1 through 9 in each cell, in such a way that:

✓ Each vertical column (shown in yellow) contains the digits from 1-9 exactly once

✓ Each horizontal row (shown in pink) contains the digits from 1-9 exactly once

✓ Each sub-grid or region (shown in green) contains the digits from 1-9 exactly once.





Instead of digits, other symbols can be used, e.g. letters, as long as there are nine different symbols.

• In each Sudoku puzzle, several digits, the "**givens**", are shown. These cannot be changed by the solver. The solver has **to fill the blank grids with digits following the rules listed above.**

- A "good" Sudoku puzzle has only **one** solution.

Solve this Sudoku puzzle.

	5	9		2		4	6	
1	2		4		3	7		8
3		4		7		9		2
	3		8	5	9		2	
6		5		4		3		7
	1		7		6		4	9
2		3		1	7		9	4
9			3		2	6		5
	7	8	9	6		2	3	

The solution to the Sudoku puzzle shown in the previous page.

7	5	9	1	2	8	4	6	3
1	2	6	4	9	3	7	5	8
3	8	4	6	7	5	9	1	2
4	3	7	8	5	9	1	2	6
6	9	5	2	4	1	3	8	7
8	1	2	7	3	6	5	4	9
2	6	3	5	1	7	8	9	4
9	4	1	3	8	2	6	7	5
5	7	8	9	6	4	2	3	1

Adapted from http://www.sudoku.ws/rules.htm

Suggested Puzzles

Matchstick Puzzles

Matchstick puzzles help to develop spatial perception, analytical skills, logical reasoning and problem solving skills. Spatial perception, or awareness, is the ability to view figures and objects within the mind and to transform the mental image so the object can be imagined from a different view point.

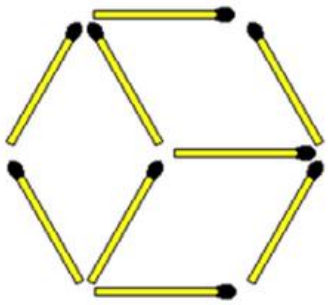
Students will be using visualization and reasoning strategies such as "***what happens if...?***" or "***working backwards"***, to solve the puzzles. The solver develops spatial awareness as (s) he experiments with moving various matchsticks. Gradually, the skill of mentally adjusting the image will be fine-tuned.

These puzzles also improve hand–eye coordination and help to increase the attention span of those solving the puzzles.

The following puzzles were taken from http://matchstickpuzzles.blogspot.com/. This site offered a large variety of graded puzzles, with challenges ranging from easy to extreme.

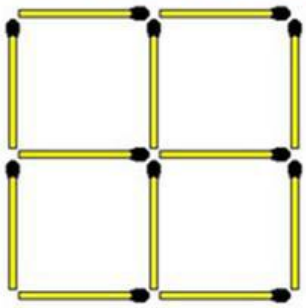
EASY LEVEL PUZZLE #1

Move 4 to create 5 triangles



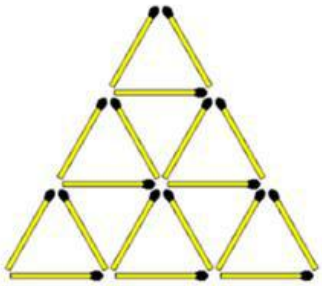
EASY LEVEL PUZZLE #2

Move 4 creating 2 squares



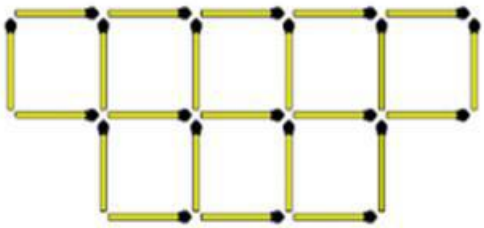
EASY LEVEL PUZZLE #3

Remove 6 leaving 3 triangles



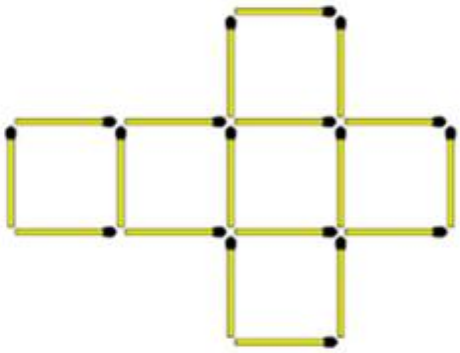
EASY LEVEL PUZZLE #4

Remove 3 sticks leaving 5 squares



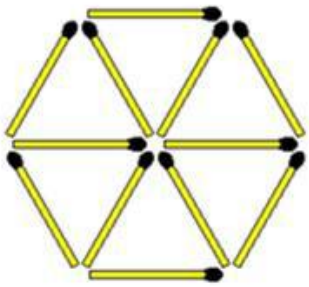
MEDIUM LEVEL PUZZLE #1

Move 4 to create 4 squares



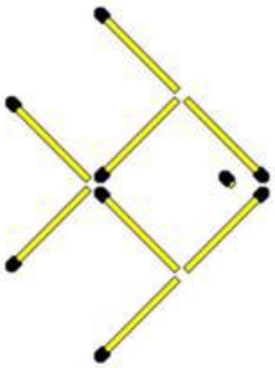
MEDIUM LEVEL PUZZLE #2

Move 2 sticks to create 6 unequal triangles



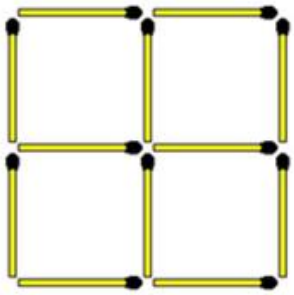
MEDIUM LEVEL PUZZLE #3

Easterly Fish, Southerly Fish



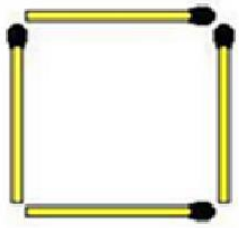
MEDIUM LEVEL PUZZLE #4

Remove 1 moving 3 forming 11 Squares



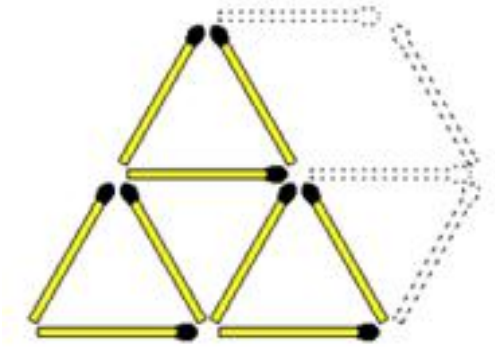
MEDIUM LEVEL PUZZLE #5

Add 5 creating 6 Squares

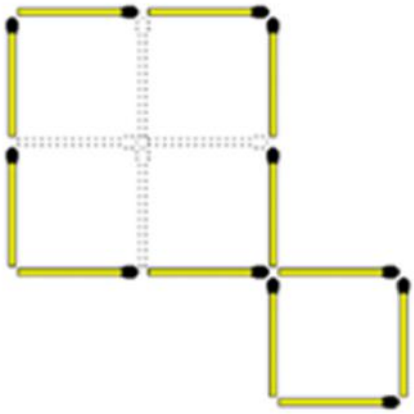


SOLUTIONS

SOLUTION EASY LEVEL PUZZLE #1

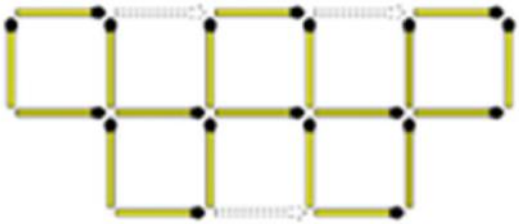


SOLUTION EASY LEVEL PUZZLE #2

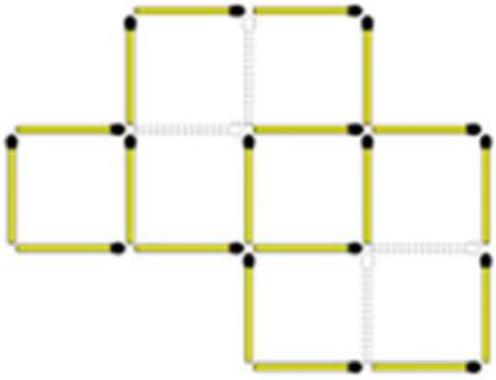


SOLUTION EASY LEVEL PUZZLE #3

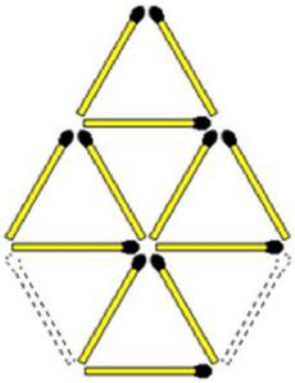
SOLUTION EASY LEVEL PUZZLE #4



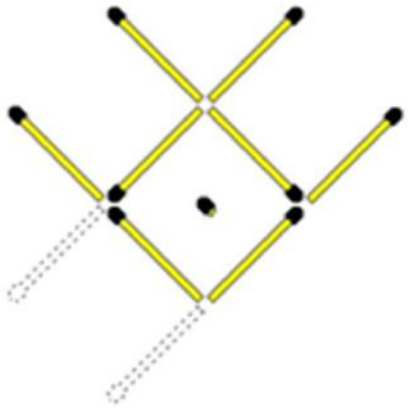
SOLUTION MEDIUM LEVEL PUZZLE #1



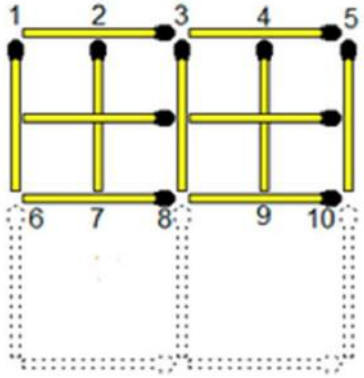
SOLUTION MEDIUM LEVEL PUZZLE #2



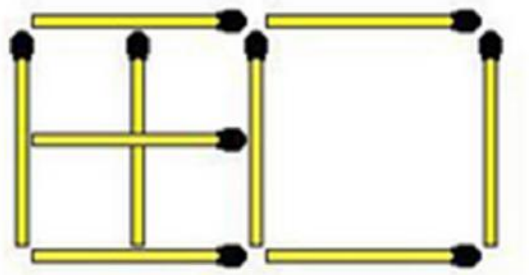
SOLUTION MEDIUM LEVEL PUZZLE #3



SOLUTION MEDIUM LEVEL PUZZLE #4



SOLUTION MEDIUM LEVEL PUZZLE #4



Suggested Activity

POLYOMINOES

Polyominoes are plane shapes made by joining squares together, where the squares are joined edge to edge so that they meet at the corners. Polyominoes are named according to the number of squares forming them:

monominoes (1 square)	dominoes (2 squares)
triominoes (3 squares)	tetrominoes (4 squares)
pentominoes (5 squares)	hexominoes (6 squares)

PENTOMINOES

A pentomino is a shape made up by joining 5 congruent squares edge to edge. There are twelve possible pentominoes, not counting rotations and reflections. Pentominoes are used in recreational mathematics for puzzles and problem solving.

Teachers can use pentominoes to introduce the important mathematical concepts of area, perimeter, symmetry and tessellation and to encourage problem solving.

In exploring the 12 combinations, students explore ideas and understandings on symmetry, rotation and reflection. Students' understanding of area and perimeter is challenged when the measurements of perimeter and area for different pentomino configurations are compared. Exploration of tessellations will allow students to discover whether all configurations can tessellate and which configurations need to be rotated.

Activity 1 – Build 12 Pentominoes

Can you explore and discover all 12 pentomino shapes? Use the grid paper to record your pentominoes.

Activity 2 - Tessellate a Plane with 1 Pentomino

Can you use only one shape as a floor tile and fit it together to cover the entire floor of a room? Use a single pentomino to tessellate the plane. (Clue - Some of the pentominoes (I), can be tessellated easily, while others (F) are more challenging).

A standard pentomino puzzle is to tile a rectangular box with the pentominoes, i.e. cover it without any overlap and without gaps. Each of the 12 pentominoes has an area of 5 unit squares, so the box must have an area of 60 units. Possible sizes are (6×10), (5×12), (4×15) and (3×20).

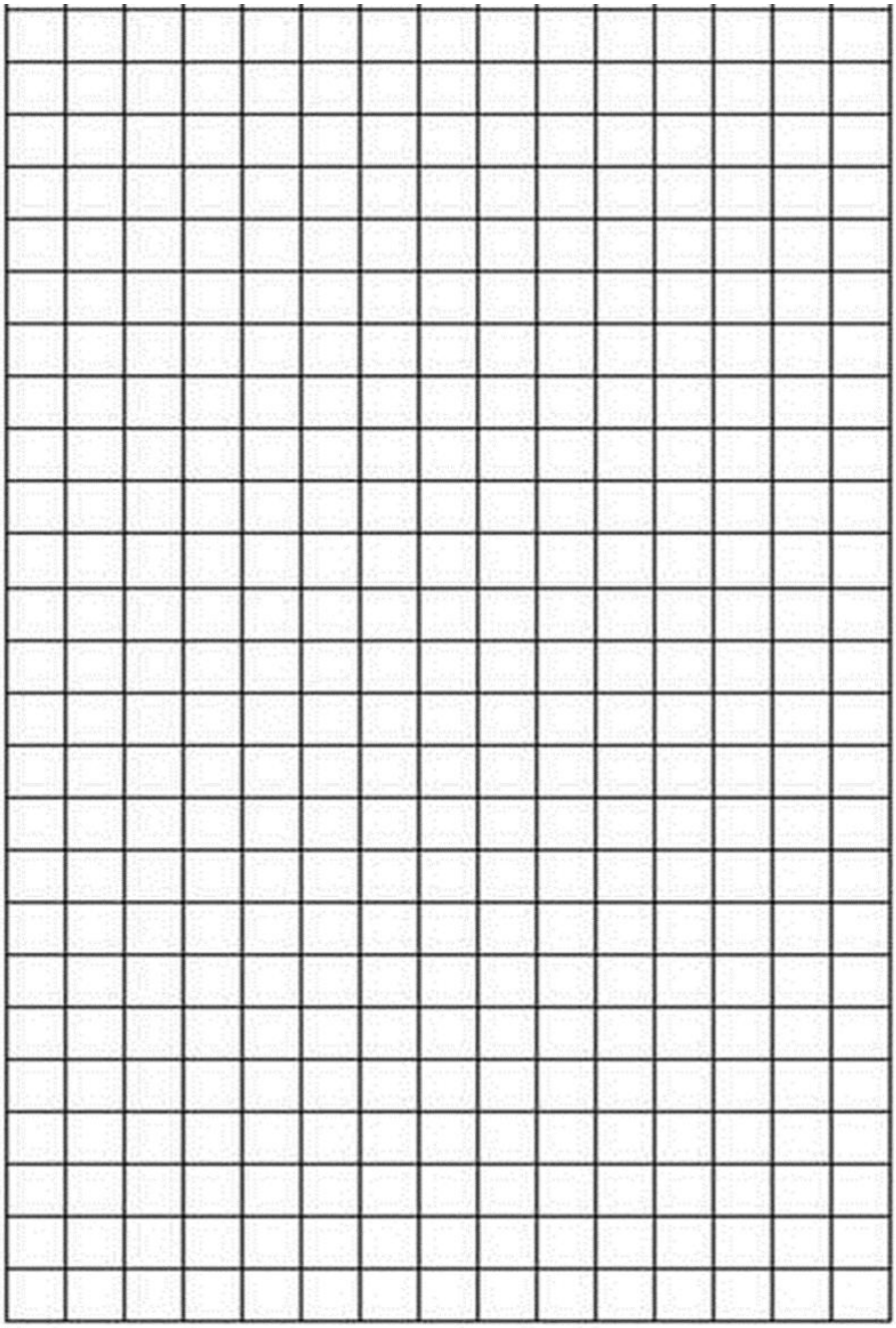
Activity 3 - Tile a Rectangular Plane

Tile the rectangular boxes (any one of the following 6×10, 5×12, 4×15 and 3×20) using each of the 12 pentominoes

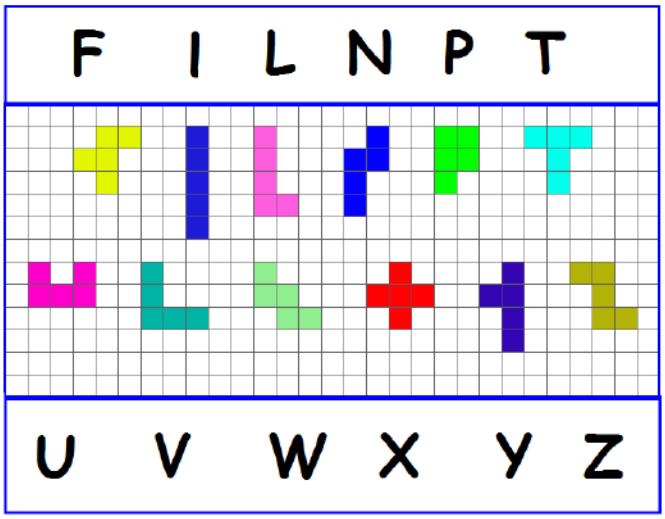
Interesting Fact

Pentominoes were featured heavily in the children's novel Chasing Vermeer, written by Blue Balliett and illustrated by Brett Helquist. The Book Trailer can be viewed at https://www.youtube.com/watch?v=qc5qNuPxxG4

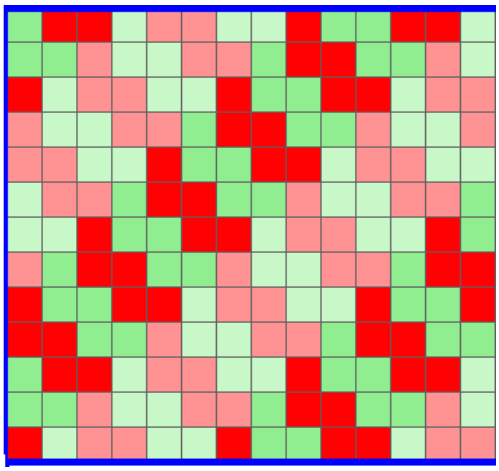
A grid for making images with the pentominoes

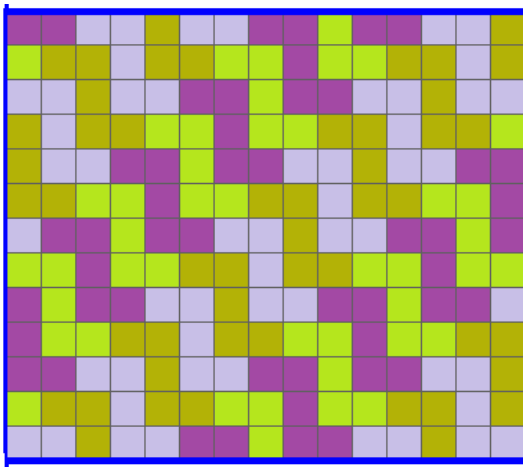


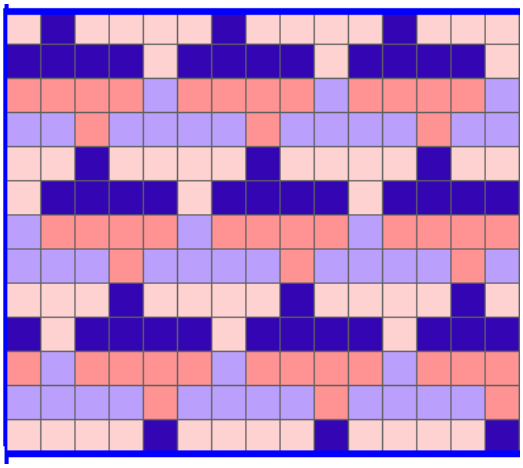
Solution - Activity 1 – 12 Pentomino Pieces



Solution - Activity 2 - Tessellate a Plane with 1 Pentomino

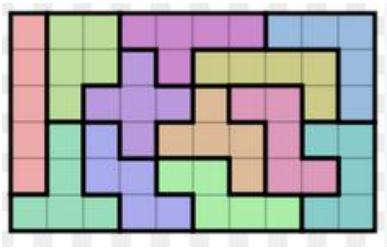






Solution Activity 3 - Tile a Rectangular Plane

The 6×10 rectangle (solved in 1960) has 2339 solutions.



The 5×12 rectangle has 1010 solutions.

The 4×15 rectangle has 368 solutions.

The 3×20 rectangle has just 2 solutions (one is shown in the figure, and the other one can be obtained from the solution shown by rotating, as a whole, the block consisting of the L, N, F, T, W, Y, and Z pentominoes).

Resources

Any set of the following:-

- Matchsticks, or toothpicks
- Straws
- Pentominoes (plastic or wooden)
- Grid paper

Further Resources

Websites

http://www.websudoku.com/

http://www.sudokukingdom.com/

http://matchstickpuzzles.blogspot.com/

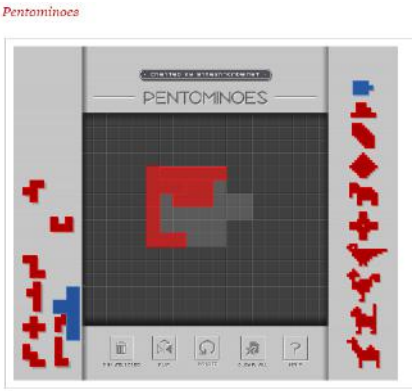
Apps

https://play.google.com/store/apps/details?id=com.klabjan.movethematchespuzzles&hl=en

Redhead Company In Australia - http://www.redheads.com.au/games.php

Turtle Diary - http://www.turtlediary.com/grade-5-games/puzzle-games/match-stick.html

Online Pentomino Game - http://www.neok12.com/games/pentominoes/pentominoes.htm



Online Tetromino Game - Tetris – http://www.tetrisfriends.com/games/Marathon/game.php

Further Reading

http://www.nytimes.com/2015/10/12/opinion/the-importance-of-recreational-math.html
The Importance of Recreational Math by Manil Suri

Problem Area 4: Multiplication Tables and Strategies

Republic Of Trinidad And Tobago, Ministry Of Education, Primary School Curriculum, Curriculum Guides, Mathematics, Infant 1 – Standard 5, Curriculum Planning and Development Division, 2013.

Standard 3

Number **Patterns**

CONTENT

1.1.5. Develop algebraic thinking (number patterns and number relationships).

SKILLS

1.2.11 Explore number patterns involving the four operations using whole numbers.

DISPOSITIONS

1.3.3 be explorative when examining patterns.

OUTCOMES

7. Demonstrate an understanding of patterns using whole numbers and involving the four operations.

ELABORATIONS

- Explore, describe and record patterns for:
- Multiplication and division facts up to the 10 times table (using concrete materials, pictorial representations, symbols, hundred chart)

Standard 3

Number Relationships

CONTENT

1.1.7. Make sense of addition, subtraction, multiplication and division number sentences involving one unknown.

SKILLS

1.2.14 Calculate the unknown in number sentences involving addition, subtraction, multiplication and division of whole numbers and involving one unknown.

DISPOSITIONS

1.3.4 Show perseverance in finding solutions to problems.

OUTCOMES

10. Explore number relationships involving the four operation using number sentences with one unknown.

ELABORATIONS

- Solve problems involving number sentences with one unknown number (represented by a symbol) e.g. 42 ÷ □ = 7; the idea of the inverse operation can be used: 7 ×? = 42, and explain reasoning. [1.1.6, 1.1.7, 1.2.14, 1.3.4]
- Solve number sentences when the unknown is on the left or right side of the equal symbol. [1.1.6, 1.1.7, 1.2.14, 1.3.4]

Standard 3

Whole Number (Operations): Multiplication and Division

CONTENT

1.1.12. Develop and apply procedures to multiply and divide whole numbers to solve problems.

1.1.13. Solve real-life problems (concrete, pictorial and symbolic modes, including money) involving multiplication and division.

1.1.14. Develop estimation skills.

1.1.15. Demonstrate an understanding of the relationship between multiplication and division.

1.1.16. Create number stories.

SKILLS

1.2.20 Develop and use the algorithm for multiplication and division of whole numbers.

1.2.21 Solve real-life problems involving multiplication (up to 2 digit by 2 digit numbers) and division (up to 4 digit divided by 1 digit).

1.2.22 Determine the reasonableness of answers by using estimation.

1.2.23 Use the inverse operation to check answers.

1.2.24 Explain or demonstrate how an answer was obtained when solving problems.

1.2.25 Create number stories involving multiplication and division and using appropriate vocabulary.

DISPOSITIONS

1.3.6 Appreciate the use of algorithms in solving problems involving the operations (multiplication and division).

OUTCOMES

16. Develop and apply procedures to multiply and divide whole numbers to solve problems.
17. Solve a variety of word problems using problem solving strategies including mental strategies.
18. Demonstrate an understanding of estimation skill.
19. Use the relationship between multiplication and division to check answers.
20. Create number stories using appropriate language.

ELABORATIONS

- Model the multiplication of whole numbers, concretely or pictorially (using an area model/arrays) and record the process (using drawings, numerals, symbols and words and the distributive property). [1.1.12, 1.2.20, 1.3.6]
- Explain through the use of words and diagrams the procedures involving multiplication using whole numbers. [1.1.12, 1.2.20, 1.3.6]
- Generalize and apply rules (algorithms) for multiplication involving whole numbers. [1.1.12, 1.2.20, 1.3.6]
- Model division of whole numbers concretely or pictorially and explain and record the process. [1.1.12, 1.2.20, 1.3.6)
- Create number stories involving multiplication and division and using appropriate language. [1.1.16, 1.2.25, 1.3.6

Standard 4

Number Concepts, Place Value and Rounding

CONTENT

1.1.2 Develop an understanding of different types of numbers.

SKILLS

1.2.2 Differentiate between (a) factors and multiples and (b) prime and composite numbers, and identify square numbers.

DISPOSITIONS

1.3.1 Display interest while engaging in activities related to number concepts.

OUTCOMES

2. Demonstrate an understanding of different types of numbers.

ELABORATIONS

- Explore factors and multiples of numbers (using resources, such as: multiplication and division tables and calculators). [1.1.2, 1.2.2, 1.3.1]
- Classify numbers as prime or composite (up to 100) by determining the number of factors. [1.1.2, 1.2.2, 1.3.1]
- Explain why the array model of a prime number has only one row. [1.1.2, 1.2.2, 1.3.1]
- Represent composite numbers as a product of their prime factors concretely, pictorially and symbolically. [1.1.2, 1.2.2, 1.3.1]
- List square numbers (up to 144). [1.1.2, 1.2.2, 1.3.1]

Standard 4

Number Patterns

CONTENT

1.1.6 Develop algebraic thinking (number patterns and number relationships).

1.1.7 Develop an understanding of different types of numbers by exploring their patterns.

SKILLS

1.2.6 Explore repeating, increasing and decreasing patterns.

1.2.7 Explore patterns involving the effects of adding or subtracting zero to/from a number and multiplying or dividing a number by one, factors and multiples of numbers, prime and composite numbers, square numbers and square roots, compatible numbers within 1 000, double and half facts, use of related facts and multiplication and related division facts up to 12 times table.

DISPOSITIONS

1.3.2 Show perseverance in finding solutions to problems that involve patterns.

1.3.3 Display perseverance while exploring properties of numbers.

OUTCOMES

5. Develop an understanding of different types of numbers by exploring their patterns.

ELABORATIONS

- Describe repeating, increasing or decreasing patterns with fractions, decimals and whole numbers resulting from addition, subtraction, multiplication and division by stating the pattern rule which includes the starting point and a description of how the pattern continues. [1.1.6, 1.1.7, 1.2.6, 1.3.2]
- Use a pattern rule to determine missing elements for a given pattern and to extend or predict subsequent elements in patterns. [1.1.6, 1.1.7, 1.1.8, 1.2.6, 1.2.7, 1.3.2]
- Describe (verbally or written) a given pattern and explain how each element differs from the proceeding one. E.g. one more, five less. [1.1.6, 1.1.7, 1.1.8, 1.2.6, 1.2.7, 1.3.2]
- Recognize when an error occurs in a pattern and explain what is wrong. [1.1.6, 1.1.7, 1.1.8, 1.2.6, 1.2.7, 1.3.2]
- Use calculators to assist in determining the pattern rule and extending patterns. [1.1.6, 1.1.7, 1.1.8, 1.2.6, 1.2.7, 1.3.2]
- Create repeating, increasing and decreasing number patterns and explain the pattern rule. [1.1.6, 1.1.7, 1.1.8, 1.2.6, 1.2.7, 1.3.2, 1.3.3]
- Explore, describe and record patterns related to the effects of adding/subtracting zero to/from a number and multiplying/dividing a number by one, factors and multiples of numbers, prime and composite numbers, square numbers and square roots, compatible numbers within 1 000, double and half facts, use of related facts and multiplication and related division facts up to 12 times table and explain generalizations about number relationships. [1.1.6, 1.1.7, 1.2.7, 1.3.2, 1.3.3]
- Solve problems involving the use of patterns. [1.1.6, 1.1.7, 1.1.8, 1.2.6, 1.2.7, 1.2.8, 1.3.2, 1.3.3]

Standard 4

Number Relationships

CONTENT

1.1.9 Explore algebraic thinking (number patterns and number relationships).

1.1.10 Make sense of number sentences involving one unknown.

SKILLS

1.2.9 Calculate the unknown in number sentences involving the four operations and explain procedures used

DISPOSITIONS

1.3.4 Show perseverance in finding solutions to problems.

OUTCOMES

7. Solve problems involving number sentences with one unknown.

ELABORATIONS

- Calculate the missing values in number sentences with one unknown involving addition, subtraction, multiplication and division of whole numbers, by using various strategies, such as guess and check, using the inverse operation and recalling memorized facts. [1.1.9, 1.1.10, 1.2.9, 1.3.4]
- Explain procedures used in solving problems. [1.1.9, 1.1.10, 1.2.9, 1.3.4]

Standard 4

Whole Number (Operations):

CONTENT

1.1.11 Create and solve problems using whole numbers involving the four operations.

1.1.12 Develop and apply mental mathematics strategies to solve problems involving whole numbers.

1.1.13 Use estimation strategies in problem solving contexts with whole numbers

SKILLS

1.2.11 Multiply 2, 3 and 4 digit numbers by 2 digit numbers

1.2.13 Investigate and apply mental mathematics strategies in solving problems.

DISPOSITIONS

1.3.5 Develop confidence in working independently in using a variety of strategies to solve problems.

1.3.5 Develop confidence in working independently in using a variety of strategies to solve problems.

OUTCOMES

8. Solve problems using whole numbers involving the four operations.
9. Demonstrate an understanding of algorithms, mental strategies and estimation

ELABORATIONS

- Create and solve problems in addition (sum less than 10 000), subtraction (minuend less than 10 000), multiplication (two, three or four-digit numbers by two-digit numbers) and division (two, three or four digits by a two-digit number) by using appropriate written algorithm and mental strategies. [1.1.11, 1.1.12, 1.2.10, 1.2.11, 1.2.12,
- Explore, describe and use a range of mental strategies and recording strategies for solving problems, including:
    - Compatible numbers within 1 000
    - Double and half facts
    - Use of related facts
    - Multiplication and related
    - Division facts up to 12 times table
    - Square numbers and square roots. [1.1.11, 1.1.12, 1.2.10, 1.2.11, 1.2.12, 1.2.13, 1.3.5]
- Use estimation strategies such as:
    - Front-end rounding (e.g. Addition: 456 + 729 is greater than 400 + 700; Subtraction: 805 – 210 is close to 800 – 200; Multiplication: the product of 13 × 25 is more than 10 × 20; Division: the quotient of 645 ÷ 15 is less than 600 ÷ 10) o Compensation (e.g. 173 + 282 + 368 + 189 + 572 is close to 200 + 300 + 400 + 200 + 500 = 1 600; because 572 is rounded down to compensate for all the other numbers being rounded up)
    - Compatible numbers (e.g. 3 248 ÷ 16; think of a basic fact that relates to the problem: 32 ÷ 16 = 2; divide: 3 200 ÷ 16 = 200; 3 248 ÷ 16 is about 200)

To check solutions to addition, subtraction, multiplication and division problems, including those involving money, and determine reasonableness of answers. [1.1.13, 1.2.14, 1.3.5]

Common Issues

Rote learning of the multiplication tables, through the repetition of multiplication facts, has been challenging for students, especially when there is a lack of sufficient opportunity to explore the effect of repeated addition. This leads to an inadequate understanding of the building of the multiplication tables. This is further compounded when the multiplication facts are presented out of context or in isolation as rote learning does not work for all students.

Some students need a variety of strategies, for example skip counting, to help in the recollection of multiplication facts. Students benefit from exercises and activities which help to make connections between multiplication and repeated addition. Activities that encourage students to develop their own meaning of the multiplication facts help students with recall and application of the tables when problem solving.

Information

The strategies shared in this booklet, provide opportunities for students to explore number patterns formed with repeated addition. Some strategies include : colouring, the drawing of lines which can be adapted for string art, finger counting, and tic tac toe grids to name a few. It is recommended to give the students the opportunity to explore the various multiplication strategies and make decisions on which one they appreciate and use more frequently.

Links to video tutorials have been included for some of the strategies shared, this can be further shared with learners to develop greater mastery and appreciation for the new methods.

Suggested Teaching Strategies

Skip Count by a Given Number Using the 0-99 Chart

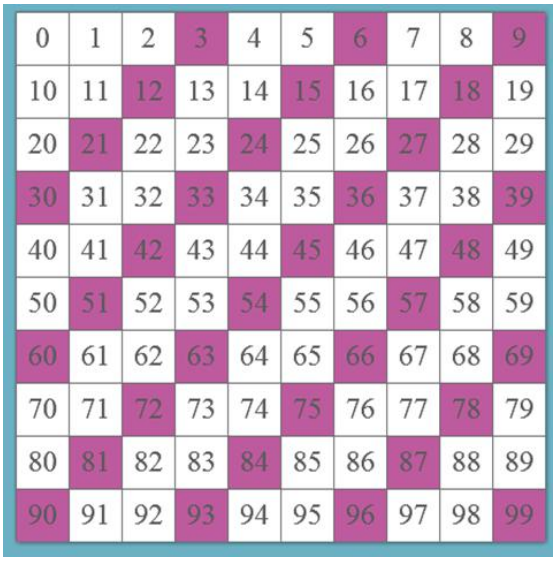
Materials:

- 0-99 chart
- coloured marker

The 0 -99 Chart helps the student to count and identify the number in the series when skip counting by a given number. To skip count, you add the same number (not 1) repeatedly.

Skip Counting By 3

- Start at 0 in the chart
- *Add 3 to get to the next number*. Point to each consecutive number, and count saying "1, 2, **3"** Colour the number 3.
- *Proceed to add 3 to get to the next number*. Point to 3, and count each consecutive number saying "1, 2, **3"** ending at 6. Colour the number 6.
- *Add 3 to get to the next number in the sequence*. Point to 6, count the next 3 consecutive numbers saying "1, 2, **3**" ending at 9. Colour 9.
- *Repeat until you reach the target number 99*
- *Observe and discuss the patterns formed*
- *With repeated practice, students will be able to transition from skip counting to the 3 times tables* and will be able to colour the numbers quickly and accurately without skip counting.

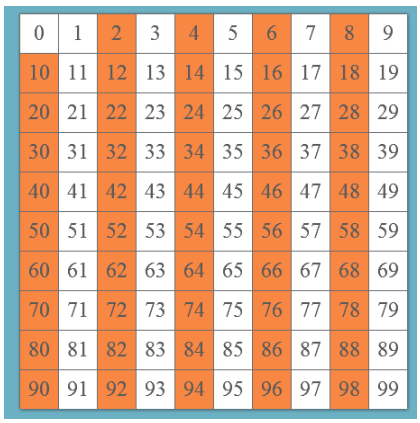


0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

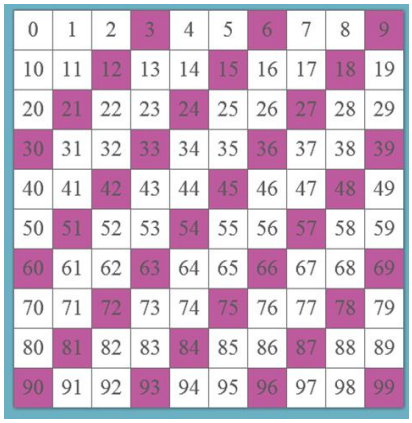
0-99 CHART

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

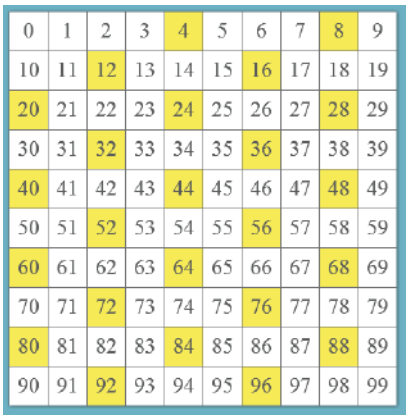
Skip Count Using the 0-99 Chart



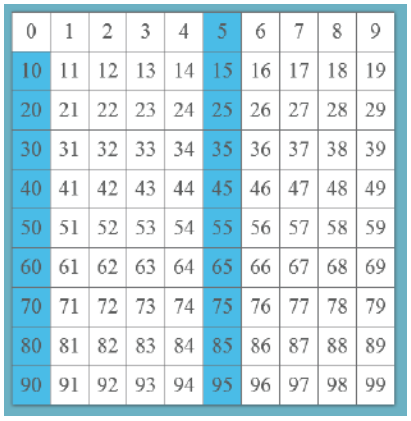
Skip Count in 2's



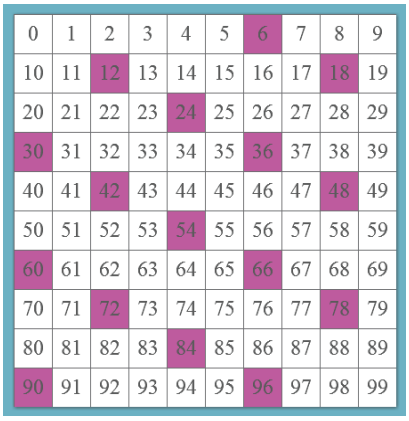
Skip Count in 3's



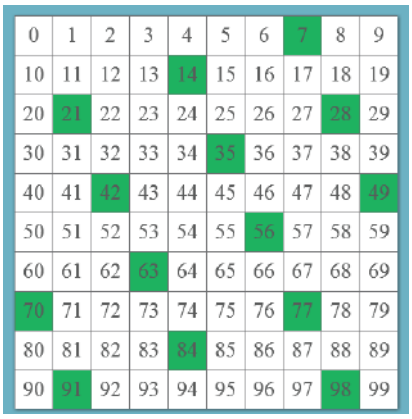
Skip Count in 4's



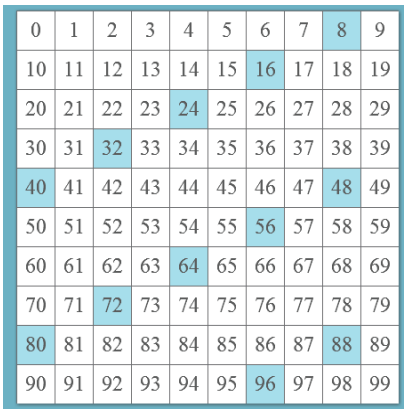
Skip Count in 5's



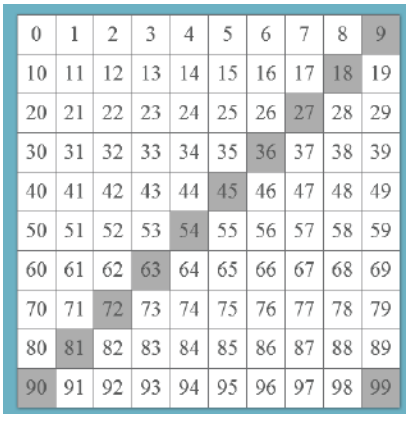
Skip Count in 6's



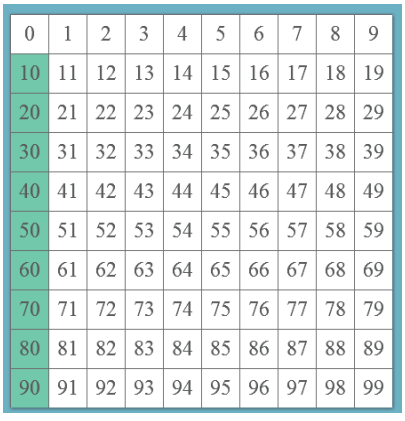
Skip Count in7's



Skip Count in 8's



Skip Count in 9's



Skip Count in 10's

Skip Count by a given number using the Waldorf Circle

Materials:

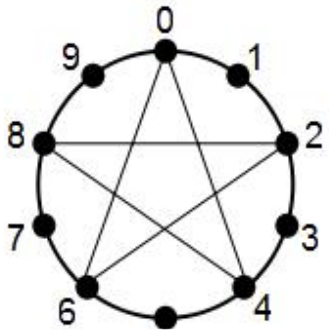
- Waldorf Circle
- Pencil / colouring pencil
- Ruler to draw straight lines

The Waldorf Circle helps the student to recognize the numbers in the multiplication tables. The numbers 0-9 are marked out on the circle. They represent the Ones digit of the numbers counted. For instance, the number 12, will be marked by the number 2, as 12 is 1 Ten and 2 Ones.

Observation are noted on the shapes formed. The shapes formed for the 2 and 8 multiplication tables are the same. So are the shapes for the 3 and 7 times tables; the 4 and 6 times tables and 1 and 9 times tables. Some shapes were formed in a clockwise direction while others were in an anti-clockwise direction.

4 Times Table

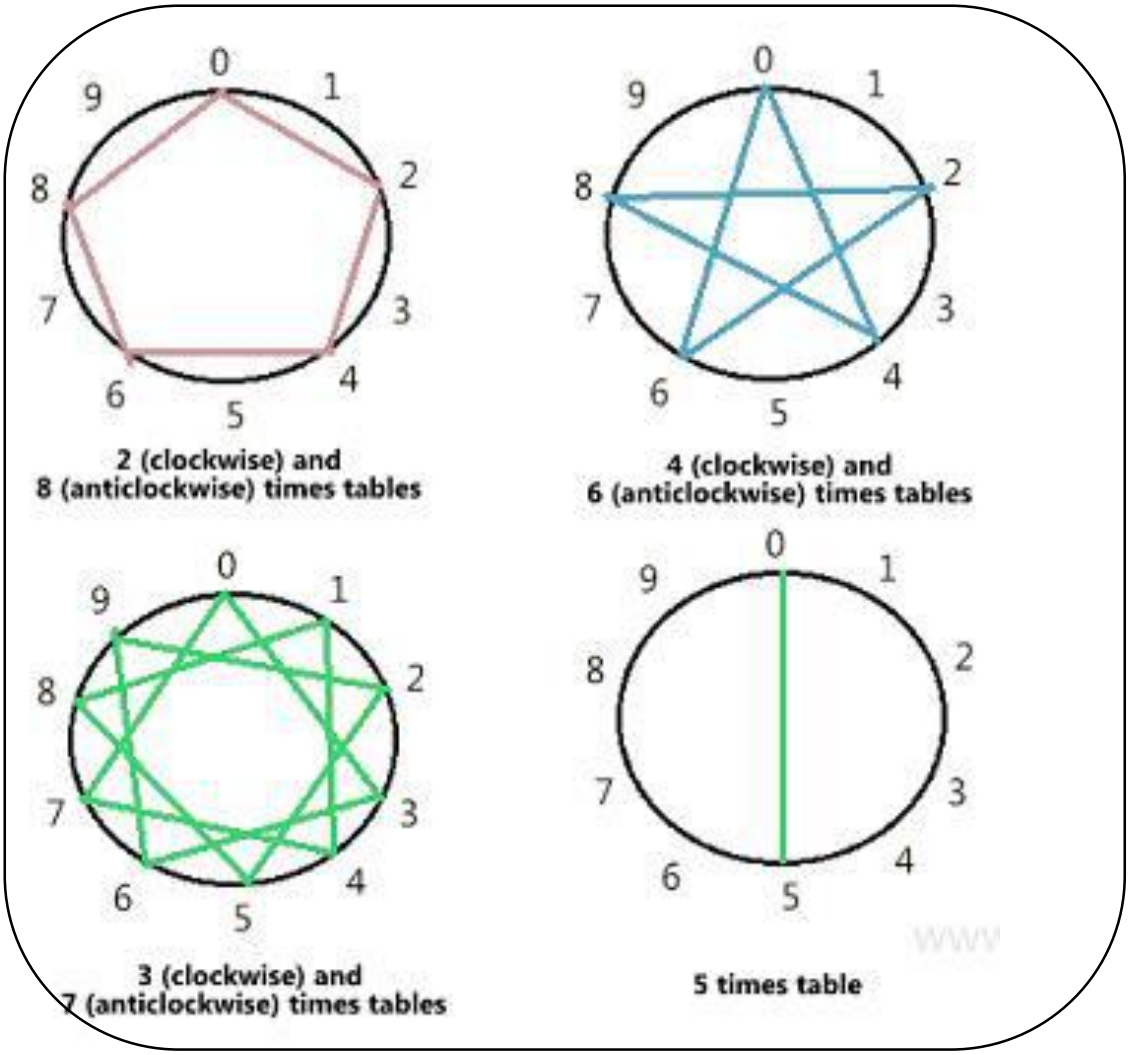
- Start at the numeral 0. Put the pencil on the dot by 0.
- Recite the first line of the 4 times table 4 x 1 = 4.
- Draw a line connecting the dots of 0 and 4.
- Recite the second line of the 4 times table 4 x 2 = 8.
- Draw a line connecting the dots of 4 and 8.
- The third line of the 4 times table is 4 x 3 = 12. The 12 is 1 Ten and 2 Ones.



Waldorf Circle showing the 4 Times Table

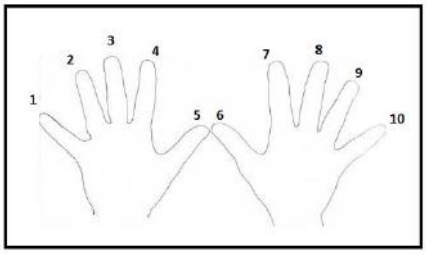
- Draw a line to the 2.
- Continue this pattern, until the 4 x 12 = 48.
- Examine the geometric pattern formed.
- Discuss the image formed (a 5 pointed star) and the frequency at which the image was made (twice by the line 4 x 10 = 40, and a partial star by the line 4 x 12 = 48).
- Repeat this activity for the numbers 1 – 9.
- Discuss the similarities and differences between the shapes formed with the different tables and the differences in the direction the lines were drawn.

GEOMETRIC SHAPES FORMED FROM THE MULTIPLICATION TABLES

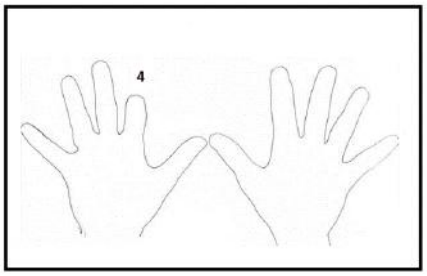


9 Times Table Using the Fingers of Your Two Hands

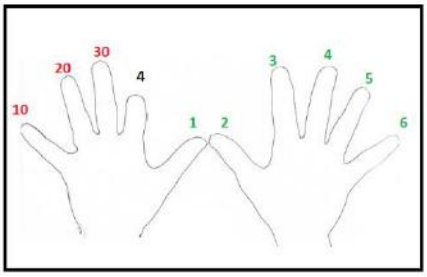
- Open both hands with the palms facing outwards and the two thumbs on the inside.
- Starting from the little finger of the left hand, count the fingers from 1-10.



- To represent any given number from a multiplication problem e.g. 9x4= ……., the number 4 will now be the 4th finger counting from the left (the index finger of the left hand). This finger will be bent inwards.



- The fingers on the <u>left of the 4th finger</u> (the folded finger), will be counted in 10s. There are three fingers, so they will be counted as "10, 20, 30".
- The fingers to the <u>right of the 4th finger</u> (the folded finger) will be counted starting from the first finger on the right "1, 2, 3, 4, 5, 6).



- The number sentence will be completed as such 9 x 4 = 36 (30 from the 3 Tens shown as the three fingers on the left and the 6 Ones from the 6 fingers on the right of the finger folded inwards)
- Note - This technique only works up to 9 x 10 = 90

9 Times Table Using the Fingers of Your Two Hands

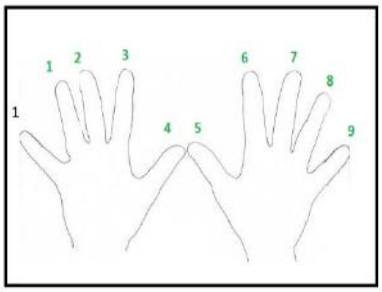


Figure 1: 9 x 1 = 9

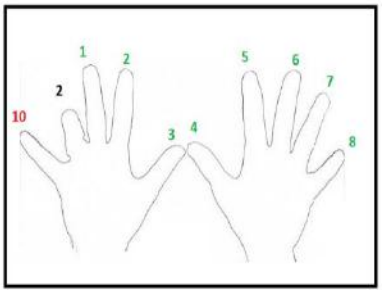


Figure 2: 9 x 2 = 18

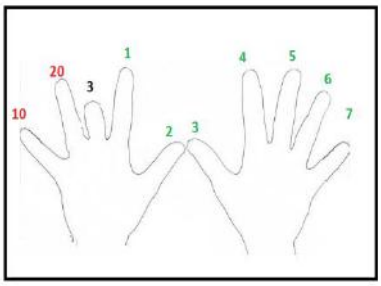


Figure 3: 9 x 3 = 27

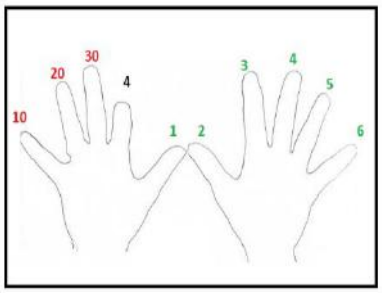


Figure 4: 9 x 4 = 36

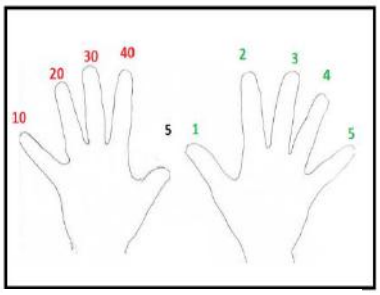


Figure 5: 9 x 5 = 45

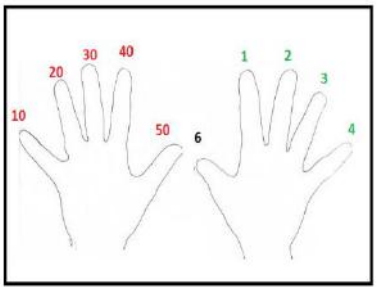


Figure 6: 9 x 6 = 54

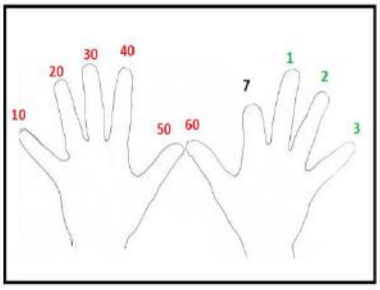


Figure 7: 9 X 7 = 63

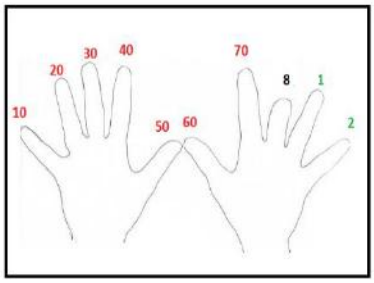


Figure 8: 9 x 8 = 72

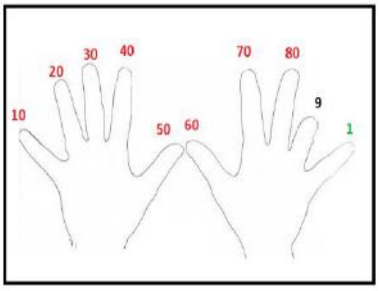


Figure 9: 9 x 9 = 81

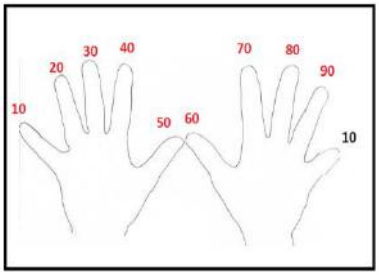


Figure 10: 9 x 10 = 90

Multiplying by Five is Just Counting by Five

E.g. 5 x 6 =?

A. Skip-count by fives up to the number.

B. Change the 5 to a 10. So it becomes 10 x 6 = 60. Find half of 60. This is 30.

C. If the factor is even, divide the factor by 2. So the multiplication problem is now

5 x 3 =? Multiply the 3 by 10 (10x3), to get the product 30.

Skip Count in 5s

<u>Bear in mind the products all end with either a 5 or a 0.</u>

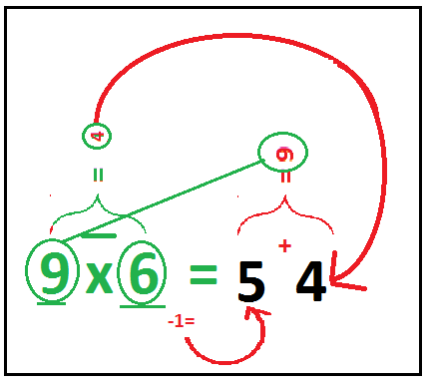
The Nine Rule

E.g. 9 x 6 =?

A. The product is 54. The digits that make up the product (5 and 4) will add up to 9.

B. The product is 54. The digit in the Tens place (5) is always one less than the number that is multiplying the 9 (1 less than 6 is 5).

C. The product is 54. The digit in the Ones place (4) is always the difference between 9 and digit in the Tens place (5), 9 minus 5 which leaves 4.

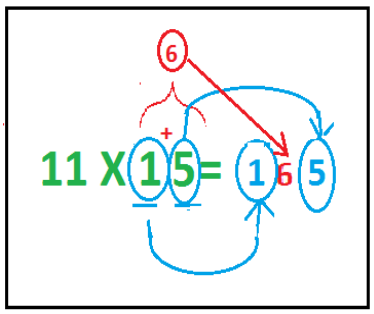


The 11 Times

A. Any number up to 9 that is multiplied by 11, the product is that number written twice. E.g. 11 x 6 = 66 *(a 6 and a 6)*

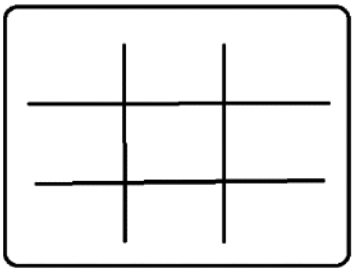
B. Any two digit number that is multiplied by 11, the product is made up of those 2 numerals that form the multiplier, written with a space in between,

E.g. 11 x 15 = 1 __ 5. The sum of the digits 1 and 5 (6) is written in between. So, 11 x 15 = 165.



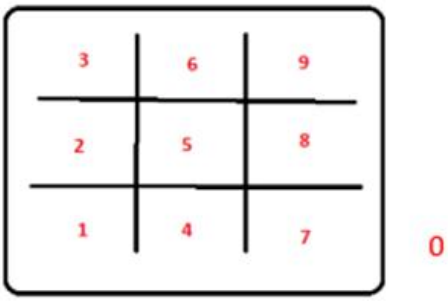
E.g. 11 x 65 = 6____ 5. But the sum of 6 and 5 is 11. Where the sum of the two digits is a two digit number, the first digit (1) is added to the first digit of the multiplier (6+1= 7). So 11 x 65 = 715.

3 Times Table with a Tick Tack Toe Grid

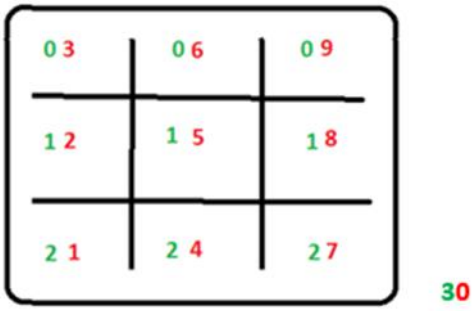


- Draw a tic-tac-toe grid.

- *Label the grid*. The numbers 1 through 9 are written starting with 1 in the box in the left column, last row. Moving upwards in the same left column, label the remaining boxes 2 and 3. 4 will be written in the box in the middle column, last row. Proceeding in an upwards direction, label with 5 and 6. In the box in the right column, lowest row, label 7. Continuing in an upwards direction, label with 8 and then 9.
- 0 will be included outside the grid on the lower right side. (*These numbers form the Ones digits of our 3 times table).*



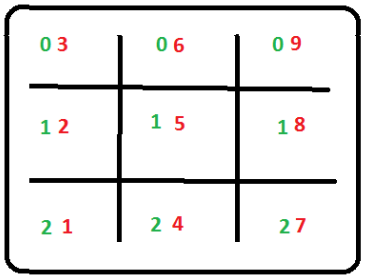
- *Write the Tens digit.* Now we will write the digits 0, 1, 2 and 3 before the numbers in a particular order.
  - 0s will be written before the numbers in the first row at the top (03, 06 and 09).
  - 1s will be placed before the numbers in the second/middle row (12, 15 and 18).
  - 2s will be placed before the numbers the third row (21, 24, and 27).
  - 3 will be placed before the number 0 on the outside of the grid. (These numbers form the Tens digits of our 3 times table).



- The answers to our 3 times table are found in this grid.
- Starting with 3x1= _____, the product (03) is in the first box in the top left hand side. The product of the second number sentence 3x2=_____will be in the box in the top row, middle column (06).
- The answers up to 3x9= _____ (27) are found in this grid. The answer to the 3x10=_____ is on the outside of the grid on the right.

6 Times Table With A Tick Tack Toe Grid.

- Start with the tick tack toe grid from the 3 times table.
- Make another Tick Tack Toe grid, writing in the numbers 1 – 9 in the same order as in the 3s.



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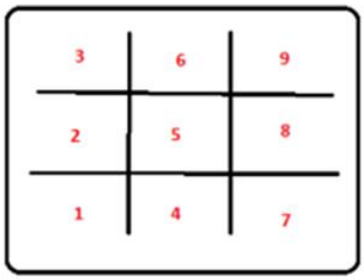
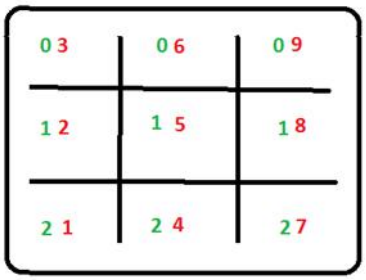


0

- The 3 times table used the numbers 0, 1, 2 and 3. The 6 times table uses the numbers 3, 4, 5 and 6 as the Tens digit for numbers in the various rows.
  - 3s will be placed as the Tens digit in the 1st row at the top of the grid.
  - 4s in the 2nd/middle row.
  - 5s in the 3rd row at the base of the grid.
  - Include the 6 before the 0 on the outside of the grid, to the lower right hand side.



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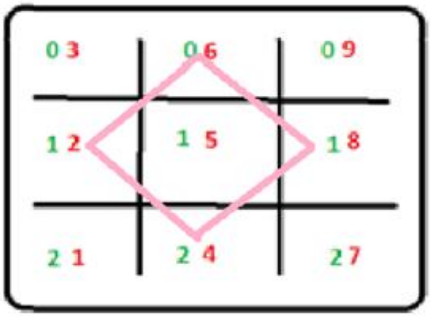
- Starting with the completed grid from the Three times table, draw a diamond. The top corner is at 06, the two middle corners will be on 12 and 18 and the lower corner will be on24. Repeat for the second completed grid. The corners will be on 36 at the top, 42 and 48 in the middle and 54 in the lowest row. Highlight the 30 and 60 numbers.



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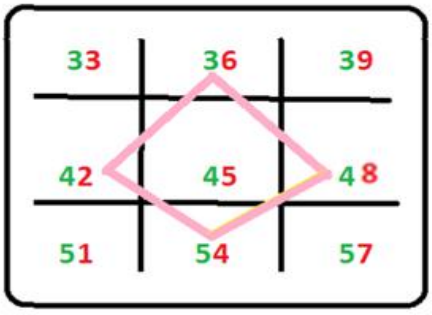
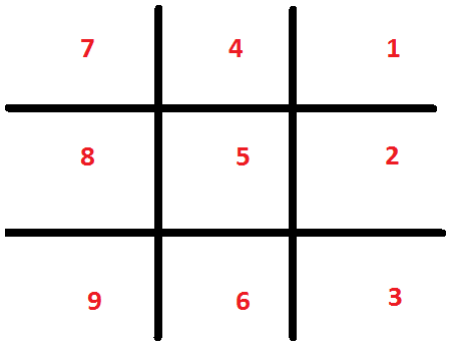


60

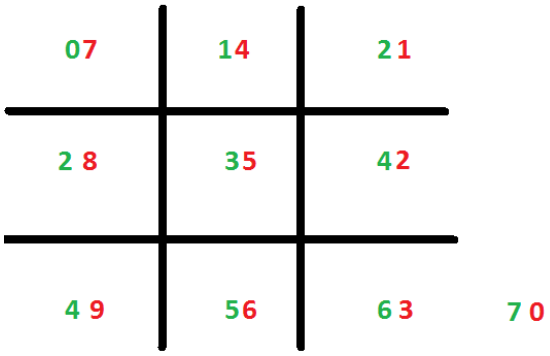
- Reading these numbers in the correct sequence will give the 6 times tables.

7 Times Table with a Tick Tack Toe Grid

- Start with the blank tick tack toe grid. The numbers 1-9 are written in, starting with 1 from the top right hand corner, moving down with 2 and 3, 4 will be the top box in the middle row, counting downwards with 5 and 6. The 7 will be the top left hand box, while 8 and 9 will be the middle and lower boxes of the column on the extreme left side.



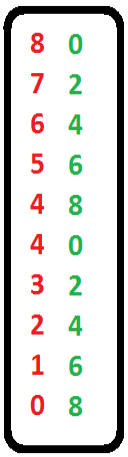
- The numbers in each row will be labelled with the Tens digit in a particular sequence.
    - The 1st row at the top will be labelled 0, 1, and 2.
    - The 2nd/ middle row will be labelled 2, 3, and 4.
    - The 3rd row at the base will be 4, 5, and 6.
    - 70 will be placed on the outside of the grid.
- The numbers in this grid, will be the numbers in the 7 times tables up to 10.



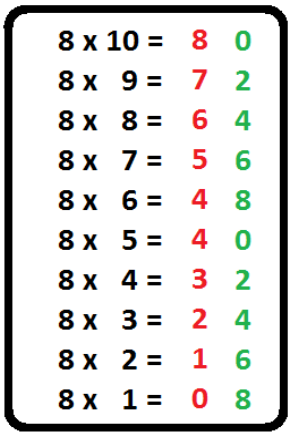
8 Times Table by Writing Numbers Vertically

- Begin by writing the numbers 8 to 0 in descending order, but include an extra 4 in the list.

- Start again at the top next to the number (8) and place the number (0) next to it. Continue writing in 2s, until you reach the number (8). This should be at the first number 4. Begin again with 0 at the next number and write, counting in 2s until you have completed the list.

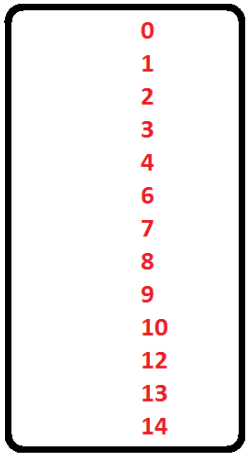


- These numbers are the products in the 8 times tables, starting from the last number, and building upwards.

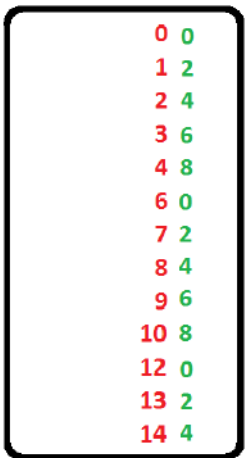


12 Times Table by writing numbers vertically

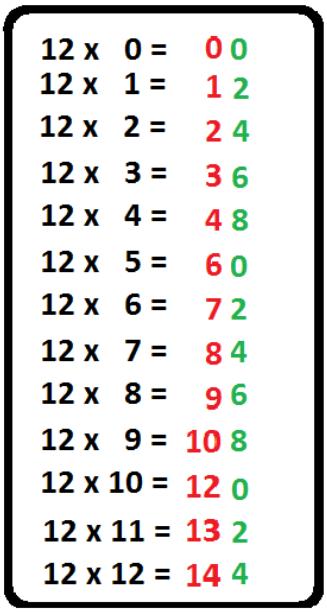
- Begin by writing the numbers 0 to 14 in ascending order, but skip out 5 and 11.



- From the first number at the top (0) write the number (0) next to it and continue counting in 2s as you go down the list. When you have reached the number 8 stop and begin again with 0 at the next number, counting in 2s until you have completed the list.



- These numbers are the products in the 12 times tables, starting from the first number sentence.

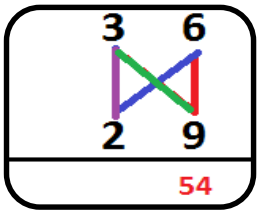


Vedic Vertical and Crosswise Math Formula

Vedic Mathematics is a system of mental calculation developed in the middle of the 20th century. This is an example of the Vertical and Crosswise formula.

Example 1: 36 x 29= _____

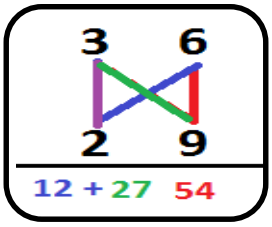
The factors are written, with the first factor (a two digit number) on top and the second factor (also a two digit number) below, so that the tens and ones are aligned vertically with each other



Step 1: Multiply The Numbers.

a) Multiply the digits on the extreme right first.

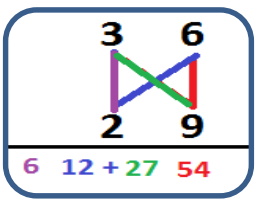
The Ones digit (6) of the first factor and the Ones digit (9) of the second factor are multiplied. Say "6 multiplied by 9 equals 54". 54 is written below, in the Ones column beneath 6 and 9.



b) Cross multiply the digits and put the resultants together.

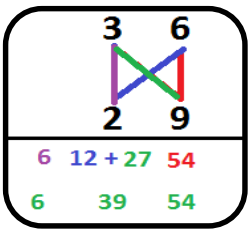
The Tens digit (3) of the first factor and the Ones digit (9) of the second factor are cross-multiplied next. Say "3 multiplied by 9 equals 27". 27 is written below, in the column beneath 3 and 2.

The Ones digit (6) of the first factor and the Tens digit of the second factor are also cross-multiplied next. Say "6 multiplied by 2 equals 12". The 12 is written in the column below 3 and 2, with an addition symbol between this product and the other product (from the cross multiplication).



c) Multiply the digits on the extreme left.

The Tens digit (3) of the first factor and the Tens digit (2) of the second factor are multiplied. Say "3 multiplied by 2 equals 6". 6 is written below, in the space to the left of the last product).

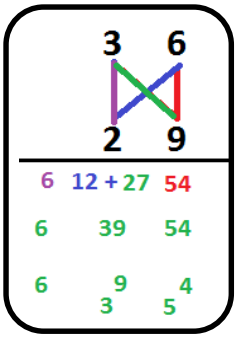


Step 2: Add The Numbers From The Cross Multiplication.

The product of 9 and 6 is 54. There is no number to add to 54, so it is written in the line below.

The two products from the cross multiplication are 12 and 27. Their sum is 39. 39 is written in the line below, to the left of the 54. The product of 3 and 2 is 6. There is no number to add to 6, so it is written in the line below as is.

Step 3: Writing The Numbers In Their Correct Form.

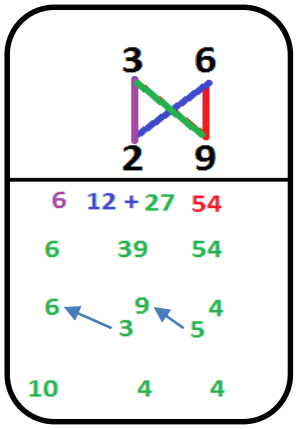


The product of 6 and 9 is 54. Write the 4 in the column below the 6 and 9, with the 5 dropped slightly lower to the left of the 4.

When one of the results contains more than 1 digit, the right hand digit is written and the left hand digit is carried over. Hence, the 39 is written with the 9 in the column below the 3 and 2, with the 3 written slightly lower.

The product of the 3 and 2, which is 6 is written in the column below 6.

Rule- the digit on the extreme left must be written as it is, whether it is a one digit or 2 digit number. Write the number on one line.



Step 4: Cross Add.

Start with the digits on the extreme right. This is a 54. The 4 (Ones) is kept and is written as it is.

The 5 will be cross added to the next column (39). The sum of 39 and 5 is 44. The 4 is kept and written as it is.

The other 4 will be cross added to the 6 from the next column. The sum of 6 and 4 is 10.

The product of 36 and 29 is 1044.

Russian Peasant Multiplication Method

The Russian Peasant Multiplication Method involves a process of halving the multiplicand (the first factor/set of factors) and doubling the multiplier (the second factor/set of factors. This reduces one factor to **powers of two**.

e.g.1: 36 x 29 = _____

Step 1: Create two Columns.

At the top of each Column, write the two sets of numbers /factors that you want to multiply.

Step 2: Divide by 2 and Multiply by 2.

Using the first number in Column 1, divide that number by 2, and keep dividing each new quotient by 2 until your final quotient is the number1- (36,18,9,4,2,1).

Rule: If there is a remainder, e.g. 9/2 is 4 and 1 remaining, ignore the remainder and write 4 in the list below the last number. Continue dividing by 2.

COLUMN 1	COLUMN 2
36	**29**
18	58
9	116
4	232
2	464
1	928

Starting with the top number in Column 2, multiply that number by 2 and keep multiplying each new product by 2, stopping only when there are the same amount of numbers as those in Column 1. (e.g. Column 1 has 6 numbers altogether, including the original number/factor, therefore Column 2 will also have 6 numbers - (29, 58, 116, 232, 4624, 928).

Step 3: Eliminate the even numbers and keep the odd numbers.

Examine the numbers from Column1; cross out all of the Even numbers from this Column and their corresponding numbers from Column 2: (e.g. ~~36 29~~ , ~~18 58~~ , ~~4 232~~ , ~~2 464~~).

~~Keep~~ the numbers from Column 1 that are Odd, and their corresponding numbers from Column 2 (e.g. 9 116 , 1 928).

Step 4: Add the numbers from the Column 2.

Add all of the numbers from Column 2 that were kept . The sum of 116 and 928 is 1044.

The product of 36 and 29 is 1044.

e.g. 2: 513 x 24= _____

This method can be used for any set of numbers being multiplied regardless of how many digits are involved. Bear in mind, the list of numbers will increase and it will take a longer time to solve.

The product of 513 and 24 is 12, 312.

COLUMN 1	COLUMN 2
513	**24**
~~256~~	~~48~~
~~128~~	~~96~~
~~64~~	~~192~~
~~32~~	~~384~~
~~16~~	~~768~~
~~8~~	~~1536~~
~~4~~	~~3072~~
~~2~~	~~6144~~
1	12288

Lattice Multiplication Method

The Lattice Multiplication Method, uses a grid or box to multiply two multi digit numbers. The number of rows and columns depends on the number of digits of the factors. The number of digits of the 1st factor, is the number of column, while the number of digits of the 2nd factor is the number of rows.

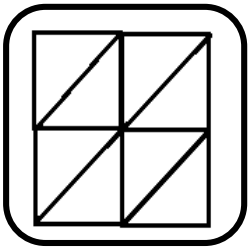
The 1st factor, the multiplicand, is written across the top row of the box and aligned with the columns.

The 2nd factor, the multiplier, is aligned with the rows, on the extreme right side of the box, the numerals moving down the side of the box The numbers are cross multiplied and written around diagonal lines. The sum of the numbers in the diagonal lines are used to form the answer of the multiplication sentence.

e.g. 1: 36 x 79 = _____

Step 1: Draw the box

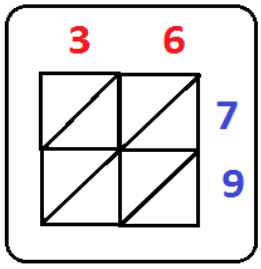
The 1st factor is a two digit number (36), so there will be two columns. The 2nd factor is a two digit number (79) so there will be two rows.



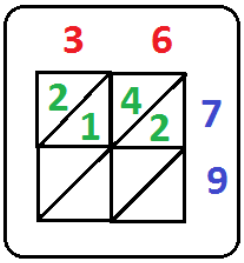
In each box, draw diagonals from the top right hand corner to the lower left corner.

Step 2: Write in the factors

The 1st factor is a two digit number (36). So on top of the first column, the 1st square will be labelled with the 3, and on top of the second column, will be labelled with the 6. The 2nd factor is 79. Outside of first row on the right hand side, the square will be labelled with the 7 and the lower right box will be labelled with the 9.



Step 3: Multiplying 36 by 7 tens

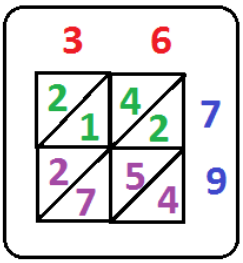


The ones digit (6) in 36 is multiplied by the tens digit (7) in 79. The product is 42 tens. We can say *"7 groups of 6s is equal to 42".* The 4 is written in the upper right hand square, above the diagonal line. The 2 is written in the same upper right hand square below the diagonal line.

The tens digit (3) in 36 is multiplied by the tens digit (7) in 79. The product is 2100. We can say *"7 groups of 3s is equal to 21".* The 2 in the 21 is written in the upper left hand square above the diagonal line. The 1 is written in the upper left hand square below the diagonal line.

All of the products were two digit numbers. If the product was a one digit number the number should be written with a 0 before it e.g. 07.

Step 4: Multiplying 36 by 9 ones

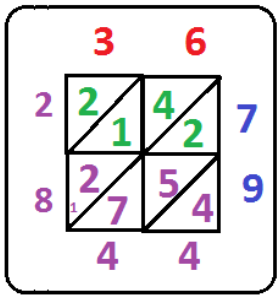


The ones digit (6) in 36 is multiplied by the ones digit (9) in 79. The product is 54 ones. We can say *"9 groups of 6s is equal to 54".* The 5 is written in the lower right hand square, above the diagonal line. The 4 is written in the same lower right hand square below the diagonal line.

The tens digit (3) in 36 is multiplied by the ones digit (9) in 79. The product is 27 tens. We can say *"9 groups of 3s is equal to 27".* The 2 in the 27 is written in the lower left hand square above the diagonal line. The 7 is written in the lower left hand square below the diagonal line.

Remember the rule if the product was a one digit number, write the number with a 0 before it, e.g. 07, as a place holder for the Tens.

Step 5: Add the diagonals



Begin adding the numbers in the diagonals.

Start with the diagonal in the lowest right hand square. 4 is the only number. It is written below the last row on the outside of the box.

The 2nd diagonal, has the numbers 7, 5 and 2. The sum is 14. The 4 is written below the last row on the outside of the box.

However, the 1 is carried over to the next diagonal. It is written smaller than the other numbers within the diagonal.

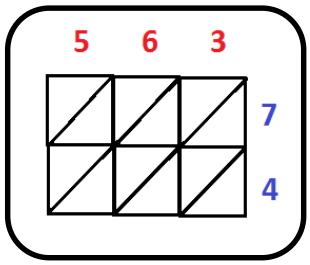
The next diagonal has the numbers 1, 2, 1 and 4. The sum 8. The 8 is written outside of the lower left hand corner, outside of the diagonal line.

The last diagonal has the number 2 alone, so 2 is written outside of that diagonal.

The final product is read, starting from the outer top left hand diagonal, reading downward and to the right. So it is read as **2 8 4 4 or 2844.**

e.g. 2: 563x 74= _____

Step 1: Draw up the box

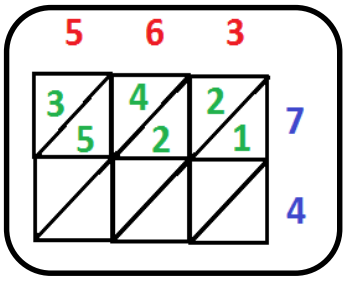


The 1st factor is a three digit number, so there will be 3 columns. The 2nd factor is a two digit number so there will be 2 rows. In each box, draw diagonals from the top right hand corner to the lower left corner.

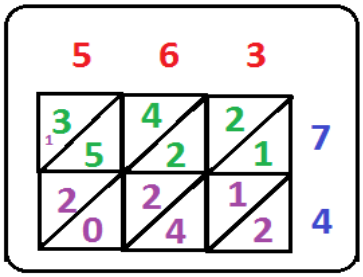
Step 2: Write in the factors

The 1st factor is a three digit number (563). So the outside of the top row, the 1st square will be labelled 5, and the 2nd square in the top row will be labelled 6 and the last square will be labelled 3. The 2nd factor is 74. Outside of the top right box, the square will be labelled with the 7 and the lower right box will be labelled with the 4.

Step 3: Multiplying 563 by 7 tens



- The ones digit (3) in 563 is multiplied by the tens digit (7) in 74. The product is 21 Tens. We can say *"7 groups of 3s is equal to 21".* The 2 is written in the upper right hand square, above the diagonal line while the 1 is written below the diagonal line.
- The tens digit (6) in 563 is multiplied by the tens digit (7) in 74. The product is 4200. We can say *"7 groups of 6s is equal to 42".* The 4 in the 42 is written in the middle column top square above the diagonal line while the 2 is written below the diagonal line.
- The hundreds digit (5) in 563 is multiplied by the tens digit (7) in 74. The product is 35 000. We can say "*7 groups of 5 is equal to 35".* The 3 is written in the upper left hand square above the diagonal line while the 5 is written below the diagonal line.
- *All of the products were two digit numbers. If the product was a one digit number the number should be written with a 0 before it, e.g. 07*

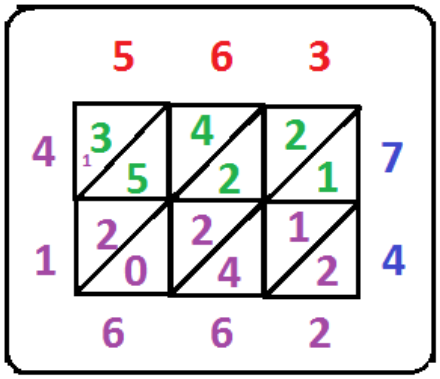


Step 4: Multiplying 563 by 4 Ones

- The ones digit (3) in 563 is multiplied by the ones digit (4) in 74. The product is 12 ones. We can say *"4 groups of 3s is equal to 12".* The 1 is written in the lower right hand square, above the diagonal line while the 2 is written below the diagonal line.
- The tens digit (6) in 563 is multiplied by the ones digit (4) in 74. The product is 240. We can say *"4 groups of 6s is equal to 24".* The 2 in the 24 is written in the middle column lower square above the diagonal line while the 4 is written below the diagonal line.

- The hundreds digit (5) in 563 is multiplied by the ones digit (4) in 74. The product is 2 000. We can say "4 *groups of 5 is equal to 20".* The 2 is written in the upper left hand square above the diagonal line while the 0 is written below the diagonal line.

Step 5: Add the diagonals



- Begin adding the numbers in the diagonals.
- Start with the diagonal in the lowest right hand square. 2 is the only number. It is written below the last row on the outside of the box.
- The 2nd diagonal, has the numbers 4, 1 and 1. The sum is 6. The 6 is written below the last row on the outside of the box.
- The next diagonal has the numbers 0, 2, 2 and 2. The sum is 6. The 6 is written outside of the lower left hand corner, outside of the box.
- The next diagonal has the numbers 2, 5 and 4. The sum is 11. 1 One is written outside that box. However, the 1 Ten is carried over to the next diagonal. It is written smaller than the other numbers.

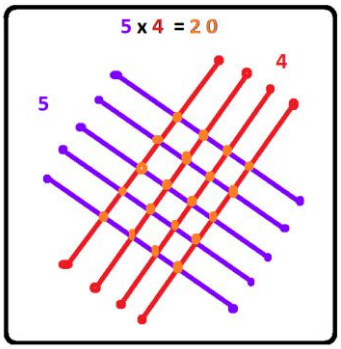
The next diagonal has the numbers 3 and 1. The sum is 4. So 4 is written outside of that box.

- The final product is read, starting from the top of the outer left hand box, reading downward and to the right. So it is read as 4 1 6 6 2 or 41, 662. **The product of 563 and 74 is: 41 662.**

Chinese Stick Multiplication

The Chinese Stick Multiplication method uses sticks to represent the digits of the two factors. The intersection of the two sets of sticks, when counted in a particular sequence, reflects the product.

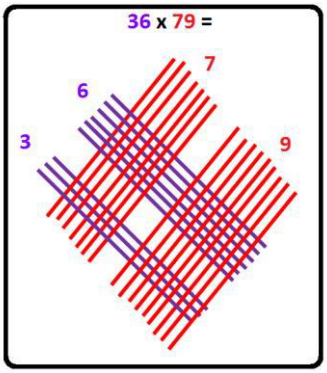
e.g.1: 5x4= ____



A 1 digit number is multiplied by a 1 digit number (1dx1d). The 5 Ones are represented by the 5 purple diagonal lines. The 4 Ones are represented by 4 red lines. The product of 5 Ones and 4 Ones are represented by the intersection of the purple and the red lines (coloured orange). The product is 20 Ones or 2 Tens and 0 Ones.

E.g. 2: 36 x 79= _____

Step 1 – Laying Out the Sticks

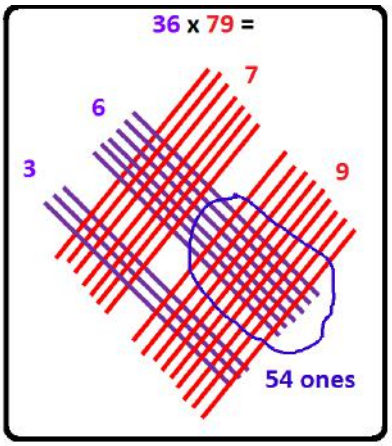


A 2 digit number is multiplied by a 2 digit number (2dx2d). The sticks are also organized to represent the two digits of each factor**.** The two factors both have Tens and Ones, so the product will most likely have a Hundred digit, a ten digit and a one digit and possibly a Thousand digit. This means we will have to look at the intersection in such a way where there is a 3 or a 4 digit answer.

- 36 has one group of 3 purple sticks (3 Tens) and one group of 6 purple sticks (6 Ones). The 3 Tens are the lower sticks while the 6 Ones are the higher sticks.
- 79 has one group of 7 red sticks (7 Tens) and one group of 9 red sticks (9 Ones). The Tens are the higher sticks while the 9 Ones are the lower sticks.

Step 2 – The Product of the Ones By Ones

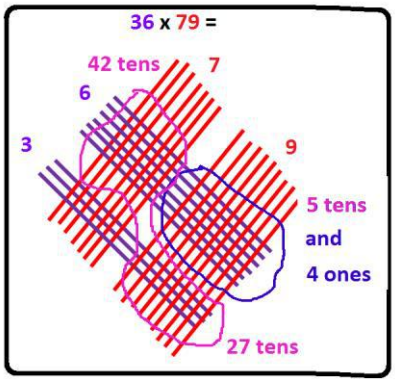
The intersections of the Ones digit (6) of 36 and the Ones digit (9) of 79 within the blue circle will give us the Ones digit of our product. Here we have 54 intersections. This is 54 Ones or 5 Tens and 4 Ones.

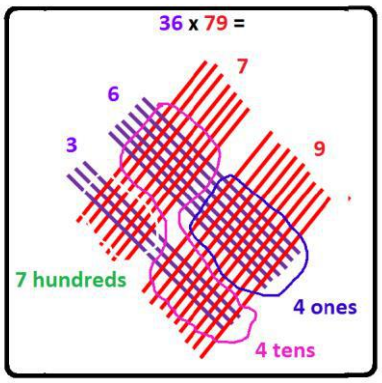


6 ones x 9 ones = 54 ones
54 ones = 5 tens and 4 ones

Step 3 – The Product of the Ones and the Tens

Here we find that the intersections of the Ones digit (6) of 36 and the Tens digit (7) of 79 within the pink circle will give 42 intersections (a product of 42 Tens). There is also the intersection of the Tens digit (3) and the Ones digit (9) of 79 (also within the pink circle) which will give 27 intersections (a product of 27 Tens). We also have to add the 5 Tens from the product of the Ones digits so we have a total of 74 Tens or 7 Hundreds and 4 Tens.

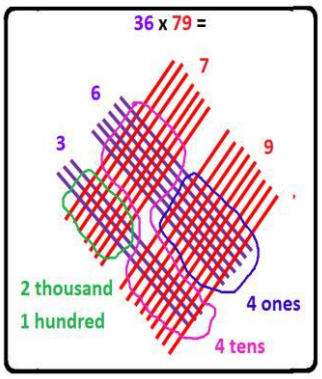




6 ones x 7 tens = 42 tens
3 tens x 9 ones = 27 tens
42 tens + 27 tens = 69 tens
69 tens + 5 tens = 74 tens
74 tens = 7 hundreds and 4 tens

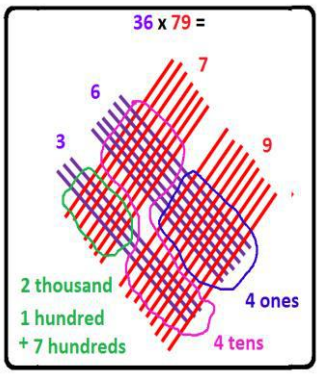
Step 4 – The Product of the Tens and Tens

We get our Hundreds digit from the intersection of the Tens digit and the Tens digit. So, within the green circle, we find that there is one intersection from the product of the Tens digit (3) of 36 and Tens digit (7) of 79, which results in 21 Hundred or 2 Thousand 1 Hundred. We have to include the carried over 7 Hundreds (from our product of Ones and Tens). We have a total of 2 Thousand 8 Hundred.



3 tens X 7 tens = 2 thousand 1 hundred

2 thousand 1 hundred + 7 hundred = 2 thousand 8 hundred



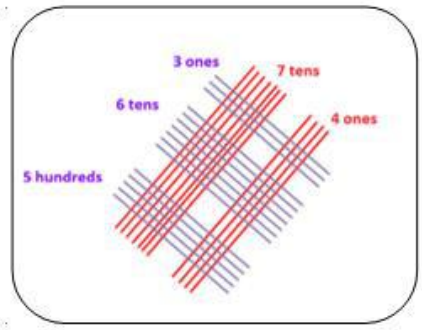
Thus, 36 x 79 is 2844.

e.g.3: 563 x 74=

Step 1 – Laying out the sticks

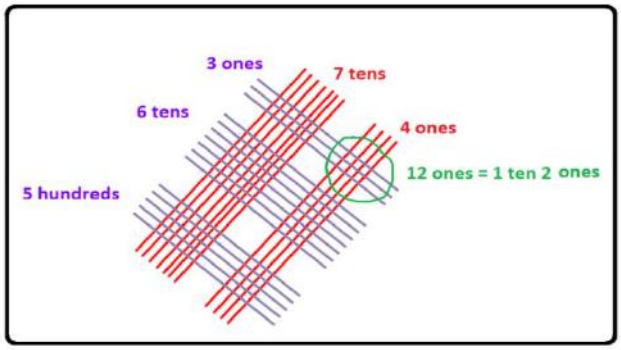
A 3 digit number is multiplied by a 2 digit number. The 5 hundred, 6 tons and 3 ones are represented by the purple diagonal lines.

The 7 tens and 4 ones are represented by the red diagonal lines.



Step 2 – The Product of the Ones By Ones

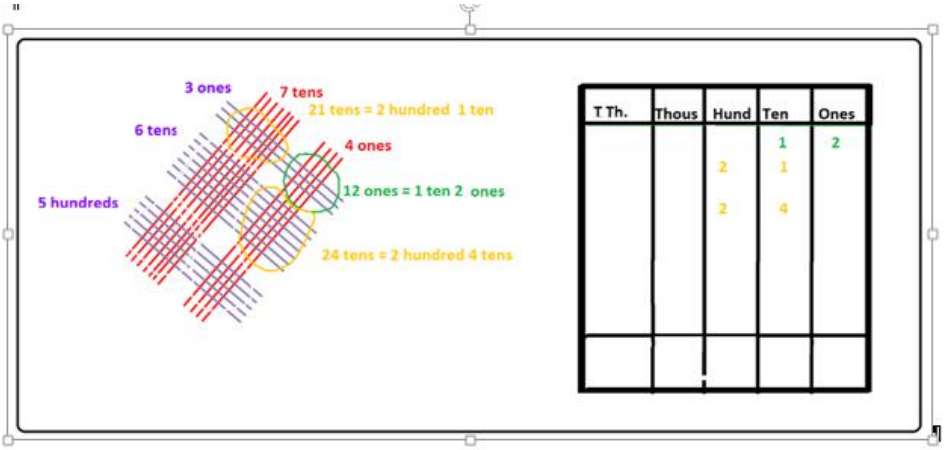
The intersections of the 3 Ones and the 4 Ones within the green circle will give us the Ones digit of our product. Here we have 12 intersections. This is 12 Ones or 1 Ten and 2 Ones.



T Th.	Thous	Hund	Ten	Ones
			1	2

Step 2 – The Product of the Ones and the Tens

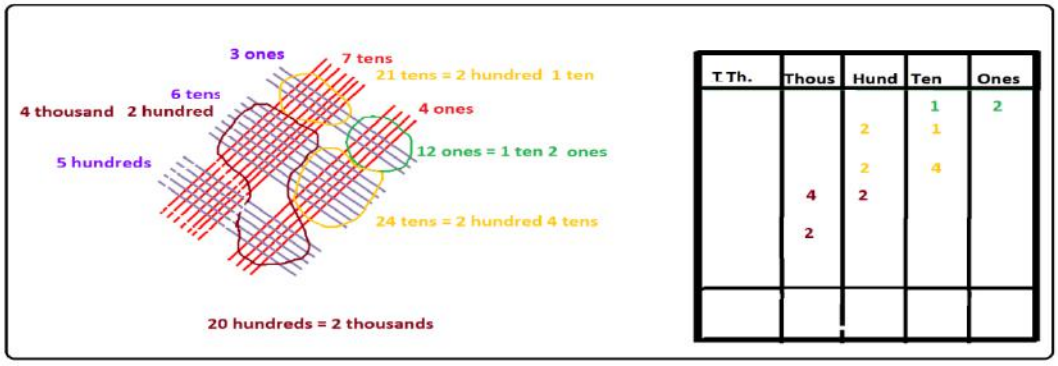
We then need to find the Tens digit of our product. This total comes from the yellow oval where the Ones digit (3) of 563 intersects with the Tens digit (7) of 74 (a product of 21 Tens) and the Tens digit (6) of 563 intersects with the Ones digit (4) of 74 (a product of 24 Tens).



T Th.	Thous	Hund	Ten	Ones
			1	2
		2	1	
		2	4	

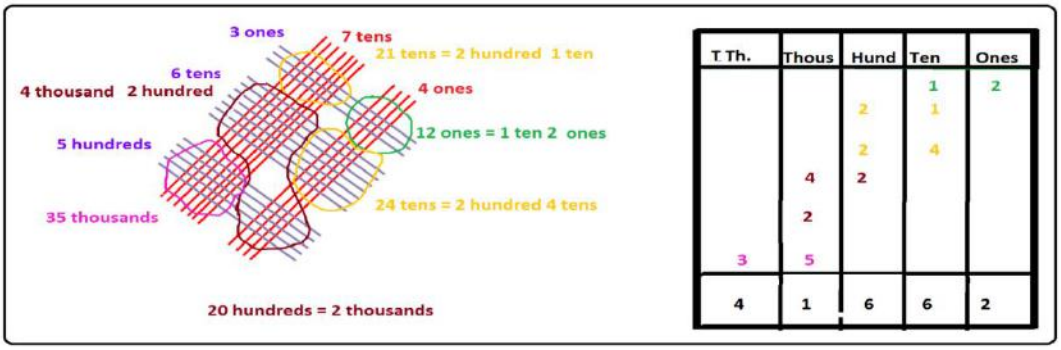
Step 3 – The Product of the Tens and Tens and Hundreds and Ones

The hundreds digit and the thousands digits of our product are found in the intersections of a tens digit (6) of 563 with a tens digit (7) of 74 (with a product of 4 thousand 2 hundred) and also a hundreds digit (5) of 563 with an ones digit (4) of 74 (with a product of 2 thousands). These intersections are found within our brown oval.



Step 4 – The Product of the Hundreds and Tens.

The thousands and tens of thousands digits of our product can be found by the intersection of the hundreds digit (5) of 563 and the tens digits (7) of 74, as shown within the purple oval. Here we get a product of 35 thousand. Our entire number sentence is complete. **The product of 563 and 74 is 41,662.**



Resources

- 0-99 chart
- Colour pencils and/or markers
- Waldorf circles
- Ruler
- 2 hands
- Tic taco toe grid
- Sticks or straws

Internet References

https://www.youtube.com/watch?v=Rhj1D_JxqV4 - 12 Times Table by writing numbers vertically

https://www.youtube.com/watch?v=-C-e33iVBTM - 8 Times Table By Writing Numbers Vertically

https://www.youtube.com/watch?v=6GqWD6i3whc - 7 Times Table With a Tick Tack Toe Grid

https://www.youtube.com/watch?v=UkEVq-uwMeE - 6 Times Table With a Tick Tack Toe Grid.

https://www.youtube.com/watch?v=8b_ghkD8jBE - 3 Times Table With a Tick Tack Toe Grid

http://www.pedagonet.com/maths/vedic.htm - Vedic Vertical and Crosswise Math Formula.

https://www.youtube.com/watch?v=NCp8S54yKqM - Fast Multiplication of 2, 3 And 4 Digit Numbers--Vedic Math's

https://www.youtube.com/watch?v=cijuPxHgZAA – Lattice Multiplication

https://www.youtube.com/watch?v=L0w8VfSUabA - Robert Brooks Multiplying 2 Digit Numbers with a 1 Digit Number using Lattice Method

https://www.youtube.com/watch?v=x2UG0YzT2UA&ebc=ANyPxKqQcsLU_mNbbkRA6A8EIxuSsdIhBklP6jjKkw3zgNWGavZelyOhEjeLawcZKsPxd585lJvEkW9EKpyFSIIKbsD0VU6h4A - Celosia Method of Multiplication.

https://www.youtube.com/watch?v=3KUckDngwUg - Celosia Method of Multiplication

https://www.youtube.com/watch?v=_AJvshZmYPs - Math Trick - Multiply Using Lines!

Problem Area 5: Elapsed Time and Resources

Republic Of Trinidad And Tobago, Ministry Of Education, Primary School Curriculum, Curriculum Guides, Mathematics, Infant 1 – Standard 5, Curriculum Planning and Development Division, 2013.

Standard 3

Measurement - Time

CONTENT

3.1.10 Develop measurement sense and apply appropriate techniques when measuring using instruments.

3.1.11 Read, interpret and record calendar dates in a variety of formats.

3.1.12 Solve problems involving time.

SKILLS

3.2.13. Read and tell time in five minute intervals on analog and digital clocks.

3.2.14. Recognize and use a.m. and p.m. in communication of information on time.

3.2.15. Tell time using a 24 hour clock.

3.2.16. Compare the duration of various events.

3.2.17. Read, interpret and record calendar dates in a variety of formats.

3.2.18. Solve real-life problems involving the measure of time.

DISPOSITIONS

3.3.5. Appreciate the functional role of the measurement of time in their everyday lives.

3.3.6. Demonstrate confidence in ones abilities to estimate and measure time.

OUTCOMES

6. Apply measurement principles, including using an instrument, to solve a wide variety of problems.

7. Read, interpret and record calendar dates in a variety of formats.

ELABORATIONS

- Tell time in five minute intervals using the digital and analog clocks. [3.1.10, 3.2.13, 3.3.5, 3.3.6]
- Match times shown on digital and analog clocks and record the time. [3.1.10, 3.2.13, 3.3.5]
- State the time after given intervals on analog and digital clocks. [3.1.10, 3.2.13, 3.3.5]
- Read and record time using the a.m. and p.m. notation and justify the need for such records. [3.1.10, 3.2.14, 3.3.5]
- Read times from a 24 hour clock and match to the analog and digital times. [3.1.10, 3.2.15, 3.3.5]
- Compare the duration of a variety of events by noting the starting and ending times and calculating the difference. [3.1.10, 3.2.16, 3.3.5]
- Use the calendar to identify and read dates. [3.1.11, 3.2.17, 3.3.5]
- Write, read and interpret dates in a variety of formats, e.g. yyyy/mm/dd, dd/mm/yyyy, March 21, 2006, dd/mm/yy. [3.1.11, 3.2.17, 3.3.5]
- Solve problems involving time. [3.1.12, 3.2.18, 3.3.6]

Standard 4

CONTENT

3.1.7 Understand that time can be quantified.

3.1.8 Solve problems in real-life situations involving time.

3.1.9 Demonstrate an understanding of time schedules.

SKILLS

3.2.12. Tell time to the minute.

3.2.13. Calculate the duration of events.

3.2.14. Estimate and verify the duration of events in minutes (up to one hour), and determine reasonableness of answer.

3.2.15. Convert hours to minutes and vice versa.

3.2.16. Solve computational and real-life problems involving hours and minutes.

3.2.17. Interpret time schedules.

DISPOSITIONS

3.3.3. Be reflective when measuring.

3.3.4. Demonstrate confidence in their abilities to estimate and measure time.

OUTCOMES

5. Accurately read and record time to the minute and solve practical problems involving time.

6. Develop an understanding of time schedules.

ELABORATIONS

- Describe time as “minutes to” or “minutes after or past” the hour and tell time to the minute. [3.1.7, 3.2.12, 3.3.3, 3.3.4]
- Match times shown on standard digital clocks, 24 hour digital clocks and analog clocks to the minute, and record time. [3.1.7, 3.2.12, 3.3.4]
- Calculate the duration of events using starting and finishing times (elapsed time). [3.1.7, 3.2.13, 3.3.3, 3.3.4]
- Estimate the duration of an event in minutes and up to one hour, verify by measuring, and determine the reasonableness of estimates. [3.1.7, 3.2.14, 3.3.3, 3.3.4]
- Convert minutes to hours and vice versa. [3.1.7, 3.2.15, 3.3.3, 3.3.4]
- Solve computational and real-life problems involving hours and minutes. [3.1.8, 3.1.9, 3.2.16, 3.2.17, 3.3.3, 3.3.4]
- Interpret simple time schedules (e.g. the calendar). [3.1.9, 3.2.17, 3.3.3, 3.3.4]

Standard 5

Measurement - Time

CONTENT

3.1.5 Solve problems in real-life contexts involving time.

SKILLS

3.2.5. Solve problems involving time and other related concepts (using proportional reasoning).

DISPOSITIONS

3.3.4. Display perseverance in solving problems related to time

OUTCOMES

4. Solve problems involving time

ELABORATIONS

- Use a number of mental and written strategies to solve familiar and unfamiliar problems involving time. [3.1.5, 3.2.5, 3.3.4]
- Use proportional reasoning to solve problems involving time and other related concepts. [3.1.5, 3.2.5, 3.3.4]

Common Issues

Solving problems involving elapsed time has proven to be challenging for students. The concept of time, the passage of time and duration of time can sometimes be abstract concepts to students especially if they have limited exposure to digital or analog clocks at home or school. Society's trend of reduced dependency on wrist watches and greater reliance on the digital time from cell phones can also impact negatively on students' ability to tell time.

Teaching time in the classroom, may be too abstract as lessons typically focus on telling time and less on understanding the passage of time. In the classroom, unless we are actually timing the students for a test, many teachers may say "In five minutes we will begin the next activity"

and in fact, a time span greater than five minutes may pass before the actual transition in activities occur.

One major obstacle to the teaching of time is the previous knowledge of the base ten system which has ten digits to show all numbers. Time is more complex, having a base of 60 for minutes and seconds, as well as 12 hour clocks with a.m. and p.m. There is confusion, even for adults in understanding the difference between 12 a.m. and 12 p.m., in relation to noon and midnight. The time number line, with markings for easy numbered intervals like 1 minute, 5 minutes, 15 minutes, help the students make the transition when solving problems with "difficult time" like 7:43 p.m.

Background Information

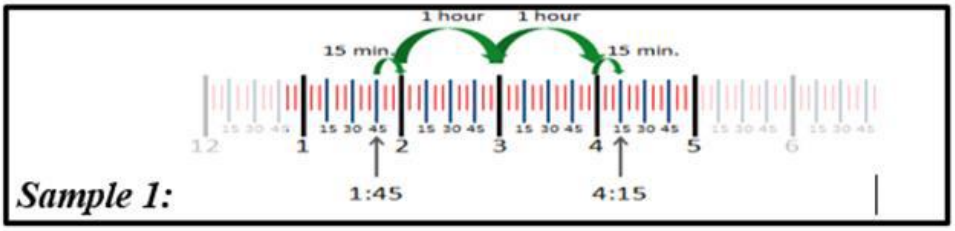
Various strategies exist for teaching elapsed time: the time number line, adaptations of the number line (with variations of the markings), the looped time number line, the T-Chart, adaptations of the T-chart with only one time given, the Z-Method and modifications of the Z-Method.

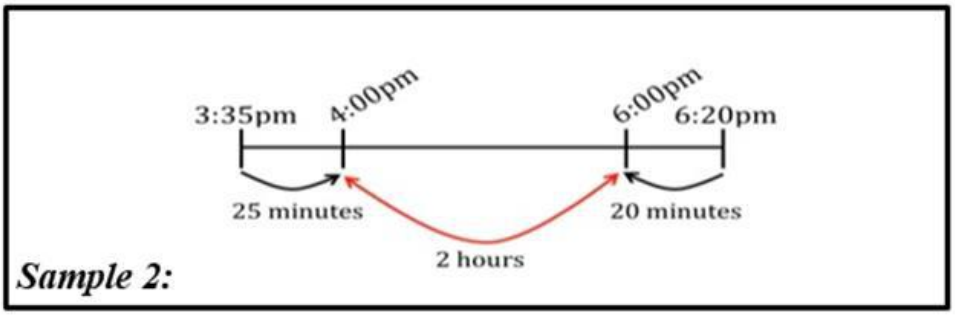
More information about the above mentioned strategies can easily be found online. One of the recommendations for overcoming math anxiety was the sharing of experiences with peers who may be facing similar situations with the hope of benefiting from the exchange of ideas. Teachers across the globe are using various online platforms to develop learning communities where personal classroom success stories and resources are shared. Similar to peer tutoring where stronger students share their understandings and experiences with weaker students, websites that encourage a community of teachers to share their classroom resources, experiences with various strategies, would be of tremendous benefit to teachers looking for new methods and technique.

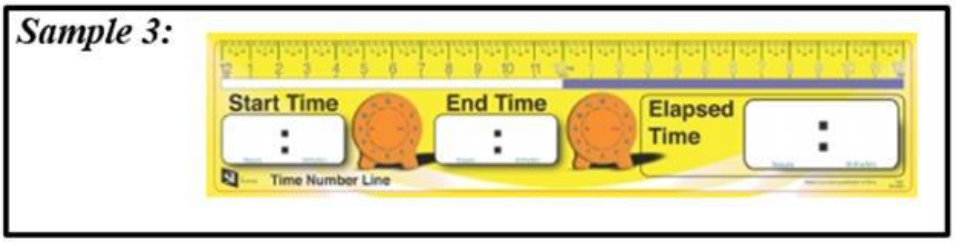
Suggested Teaching Strategies

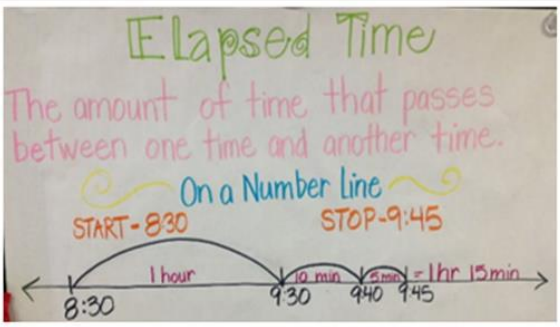
Strategy 1 - Use a Time number line

- Put in bench marks – hour, half an hour, 15 mins., 5 mins., 1 min.
- State a.m. or p.m.
- Use a printed strip and stick the two ends together to form a loop moving from 1 to 12.

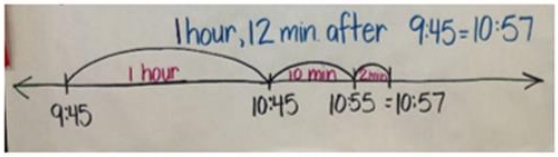




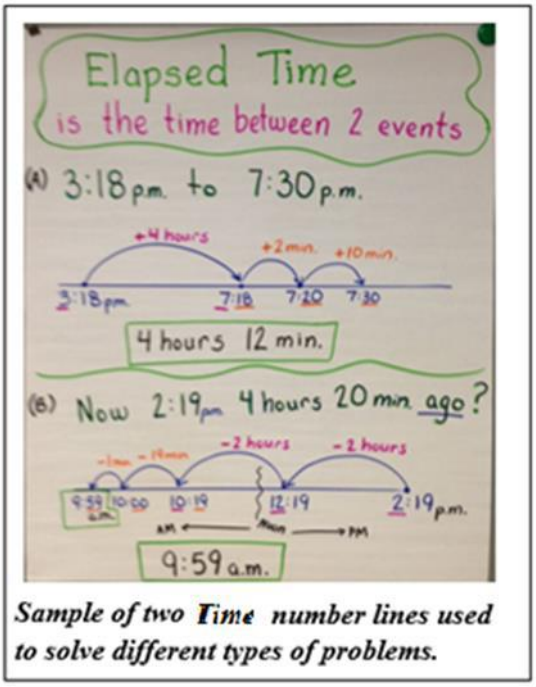




Sample of a Time number line when given the start and stop times.



Sample of a Time number line when given one of the times and the elapsed time.



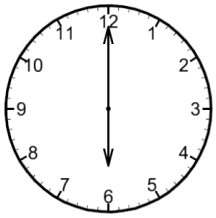
Sample of two Time number lines used to solve different types of problems.

Problems to solve.

1) *A movie started at the time shown on the right.*

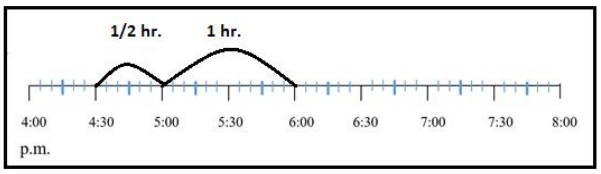
a) *At what time did the movie begin?*

b) *Patrick left home 1 ½ hrs. before the movie started. What time did he leave home?*



Solution using the number line.

- Mark the starting point (6:00 p.m.).
- Subtract the 1st benchmark (-1 hr.) by jumping back by 1 hr. (5:00 p.m.).
- Subtract the 2nd benchmark (- ½ hr.) by jumping back by ½ hr. (4:30 p.m.).

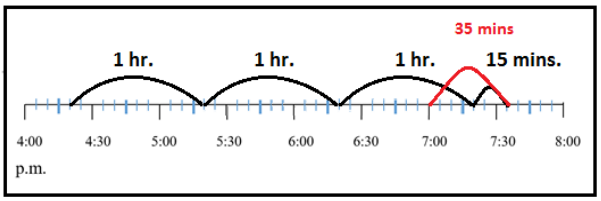


Patrick left home at 4:30 p.m.

2) *Albert started a task at 4:20 p.m. He took 3 hrs. 15 mins to complete the task. Janice took 35 mins. less than Albert. What time did Janice finish the work?*

Solution using the number line

- Mark the starting point (4:20 p.m.)
- Add the 1st benchmark (+1 hr.) by jumping forward by 1 hr. (5:20 p.m.)
- Add the 2nd benchmark (+ 1 hr.) by jumping forward by 1 hr. (6:20 p.m.).



- Add the 4th benchmark (+ 1 hr.) by jumping forward by 1 hr. (7:20 p.m.).
- Add the 5th benchmark (+ 15 mins.) by jumping forward by 15 mins. (7:35 p.m.).

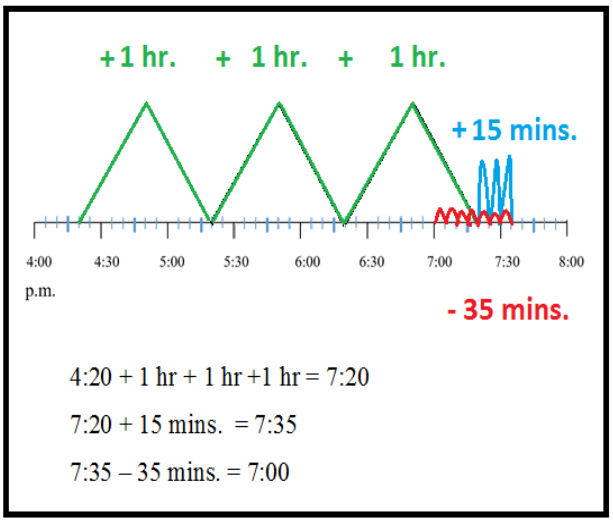
Albert completed his task at 7:35 p.m.

- Mark the starting point (7:35 p.m.).
- Subtract the 1st benchmark (-35 mins.) by jumping backwards by 35 mins. (7:00 p.m.).

Janice completed her task at 7:00 p.m.

Use a systematic coding with the Time number line.

- E.g. tall triangles / green mountains for 1 hr. intervals.
- Blue hills for 5 mins. intervals.
- Orange mounds / pebbles for 1min. intervals.

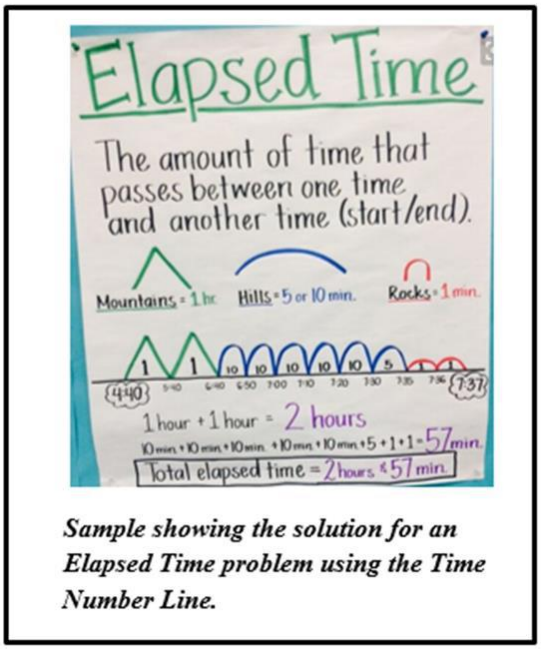


Explain using number sentences

4:20 + 1 hr. + 1 hr. +1 hr. = 7:20

7:20 + 15 mins. = 7:35.

7:35 – 35 mins. = 7:00 p.m.

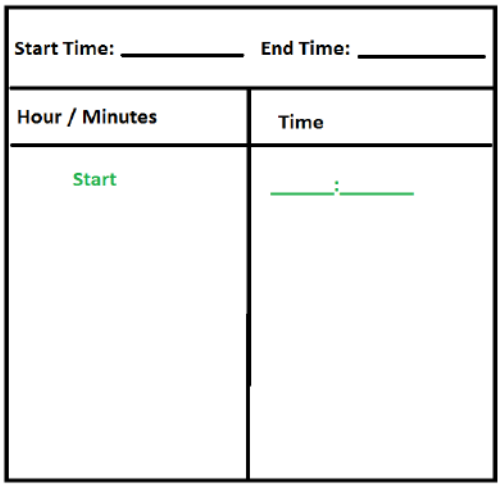


Sample showing the solution for an Elapsed Time problem using the Time Number Line.

Strategy 2 - Use a T Chart

Tips on using the T Chart to find the elapsed time, when given both times.

- State the Start and End times at the top. (It is important to remember when the end time is.)
- Draw the T Chart, and label the left column with Hour/Minutes and the right column with Time.
- First row – in the left column, under the Hour/ Minutes, write the word *START*. In the right column, under Time, write the start time.
- Under the left column, add the benchmarks of 1hr., 30 mins., 15 mins., 10 mins., 5 mins. and / or 1 min. systematically. As each benchmark is written on the left, add to give the new time on the right.



- Keep adding incremental benchmarks until the end time is reached.

Figure showing Elapsed Time with a T Chart with Time in the column on the right.

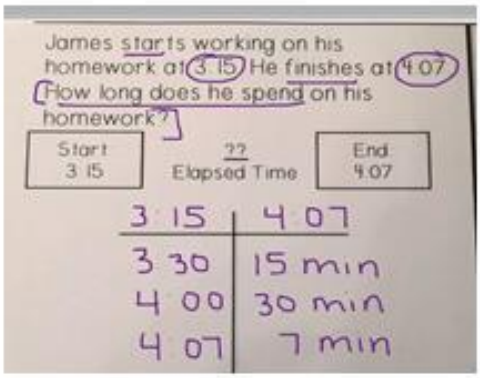
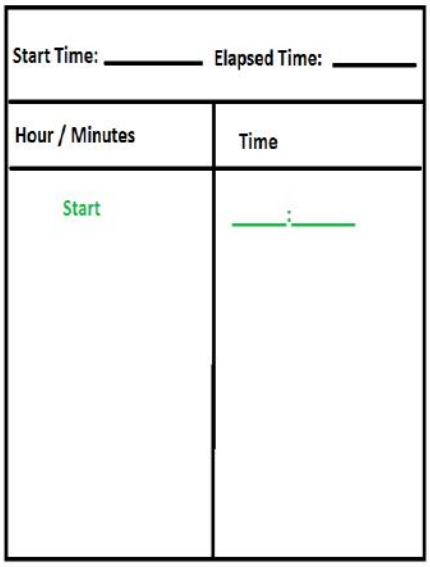


Figure showing Elapsed Time with a T Chart with Time in the column on the right.

Strategy 2 - T Charts (cont'd)

Tips on using the T Chart when given one time and the elapsed time.

- State the Start and Elapsed time at the top.
- Draw the T Chart and label the left column with Hour/Minutes and the right column with Time.
- First row – in the left column, under the Hour/ Minutes, write the word *START*. In the right column, under Time, write the start time.
- Under the left column, add the benchmarks of 1hr., 30 mins., 15 mins., 10 mins., 5 mins. and / or 1 min. which all add up to the make the Elapsed time. Next, add each benchmark to the time, in the column on the right, until all of the elapsed time is added.
- The final time is the time after the elapsed time has passed.



Given one of the times and the elapsed time, required to find the other time.

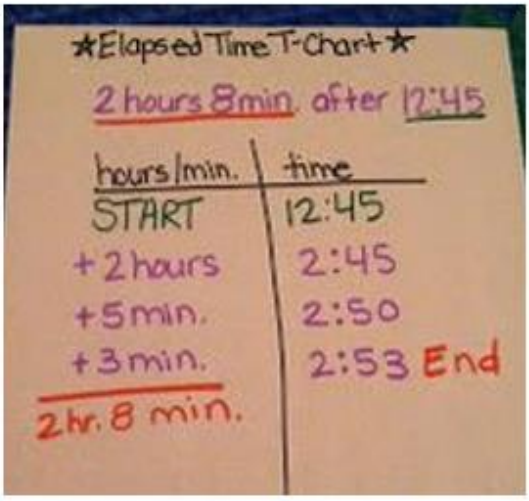


Figure showing Elapsed Time with a T Chart with Time on the right

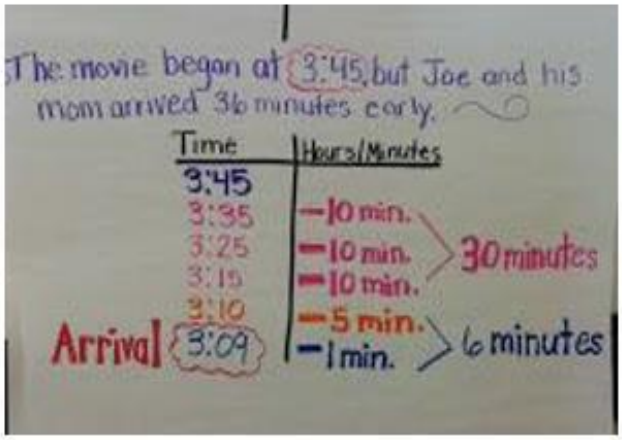
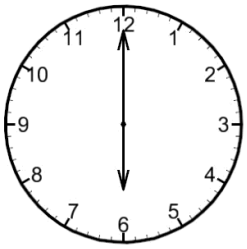


Figure showing Elapsed Time with a T Chart with Time on the left.

Problems to solve.



1 A movie started at the time shown on the right.

a) At what time did the movie begin?

b) Patrick left home 1 ½ hrs. before the movie started. What time did he leave home?

Solution using the T Chart

- Fill the blanks.
- Subtract the 1st benchmark (1hr.)
- Find the new time.
- Subtract the 2nd benchmark.
- Find the new time

Patrick left home at 4:30 p.m.

Start Time: 6:00	Elapsed Time: 1 1/2 hrs before
Hour / Minutes	**Time**
Start	6 : 00
- 1 hr.	5 : 00
- 1/2 hr.	4 : 30

2 Albert started a task at 4:20 p.m. He took 3 hrs. 15 mins to complete the task. Janice took 35 mins. less than Albert. What time did Janice finish the work?

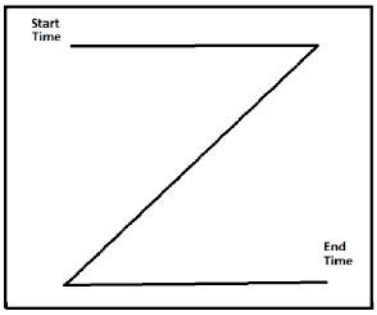
Solution using the T Chart

- Fill the blanks, stating Albert's start time and the Elapsed Time.
- Add the 1st benchmark (+ 1hr.).
- Find the new time (5:20 p.m.).
- Add the 2nd benchmark (+ 1 hr.).
- Find the new time (6:20 p.m.).
- Add the 3rd benchmark (+ 1 hr.).
- Find the new time (7:20 p.m.).
- Add the 4th benchmark (+ 15 mins.).
- Find the new time (7:35 p.m.).

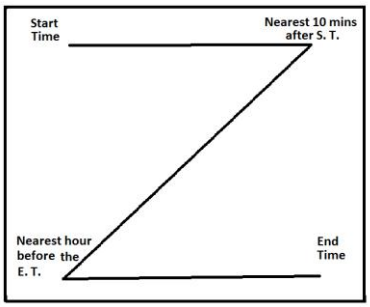
Albert Start Time: 4:20 pm	Elapsed Time 3 hrs. 15 mins
Hour / Minutes	**Time**
Start	4 : 20
+ 1 hr.	5 : 20
+ 1 hr.	6 : 20
+ 1 hr.	7 : 20
+ 15 mins.	7 : 35

Start Time: 7 : 35	Elapsed Time 35 mins less
Hour / Minutes	**Time**
Start	7 : 35
- 35 mins	7 : 00

Strategy 2 A–Z Method



- First draw a letter Z.
- At the starting point of the letter Z (top left corner), write the start time. The end point of the letter Z (bottom right corner), write the end time.

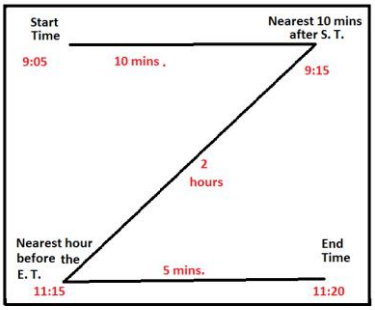


- After the first horizontal line / stroke of the letter Z, at the start of the diagonal line of the letter Z) write time to the nearest 10 minutes from /after the start time.
- At the end of the diagonal line, write the nearest hour before the end time.
- Add the different values.

Problem to solve.

A movie started at 9:05 p.m. and ended at 11:20 p.m. How long was the movie?

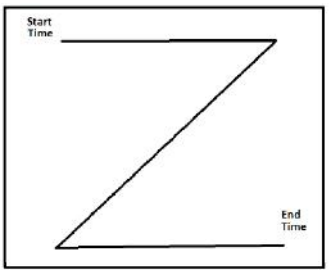
Solution using the Z Method



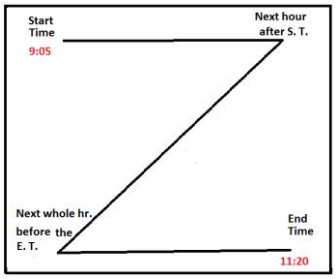
- Write the start and end times.
- Calculate the time to the nearest 10 minutes after the start time (9:05 +10 mins.= 9:15 p.m.).
- Calculate the nearest hour from the 9:15, before the end time (11:15 p.m.).
- Add the different values (2 hrs. + 10 mins. + 5 mins. = 2 hrs. and 15 mins).

The movie was 2hrs. and 15 mins. long.

Strategy 2 B–Z Method



- First draw a letter Z.
- The starting point of the letter Z (top left corner), write the start time. The end point of the letter Z (bottom right corner), write the end time.

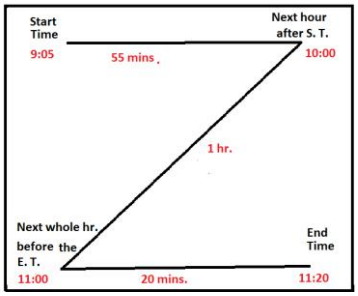


- After the first horizontal line of the letter Z, at the start of the diagonal line of the letter Z), place the benchmark times by writing the time to the next whole hour after the start time.
- At the end of the diagonal line, write the time to the next whole hour before the end time.
- Find the differences between the time from the starting point and the time of the first whole hour after the start. Label this in minutes.
- Find the difference from the two whole hrs. Label this in hours.
- Find the difference from the last hour before and end time. Label this in minutes.
- Add the two sets of minutes first and then the hours to find the duration.

Problem to solve.

A movie started at 9:05 p.m. and ended at 11:20 p.m. How long was the movie?

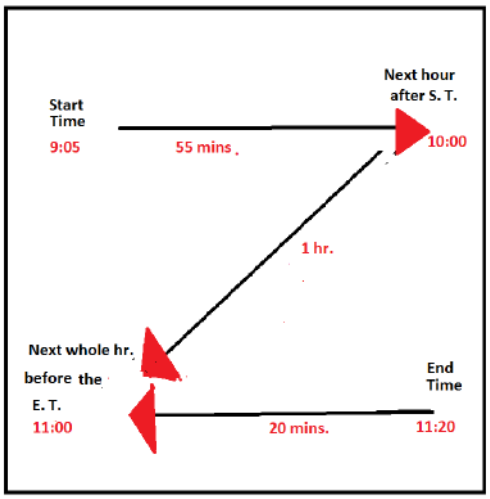
Solution using the Z Method

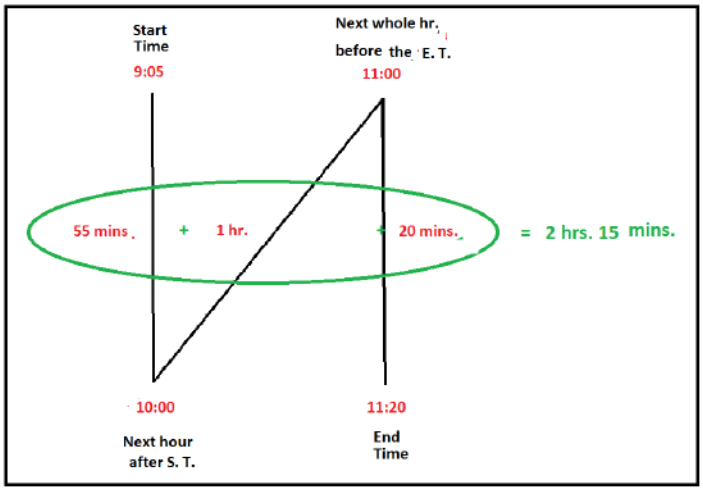


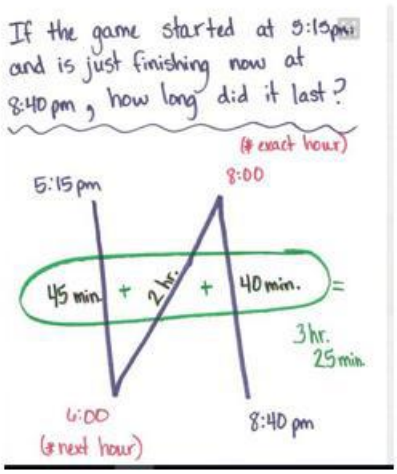
- Write the start and end times.
- Calculate the time to the nearest benchmark hour after the start time (10:00 p.m.).
- Calculate the nearest whole hour before the end time (11:00 p.m.).
- Calculate the difference between the times from the starting point (9:05 p.m.) and the first whole hour (10:00 p.m.) after the start (55 mins.)
- Calculate the difference from the two whole hrs. (10:00 p.m. to 11:00 p.m. is 1 hr.)

- Calculate the difference from the next whole hour before (11:00) and end time (11:20 p.m.). The difference is 20 mins.
- Add the minutes first (55 mins. and 20 mins. make 75 mins., which is equal to 1 hr. and 15 mins.). Then add the hour(s) to find the entire duration (1 hr. + 1 hr. +15 mins.)
- The sum of 2 hrs. and 15 mins is the answer.
  The movie was 2hrs. and 15 mins long.

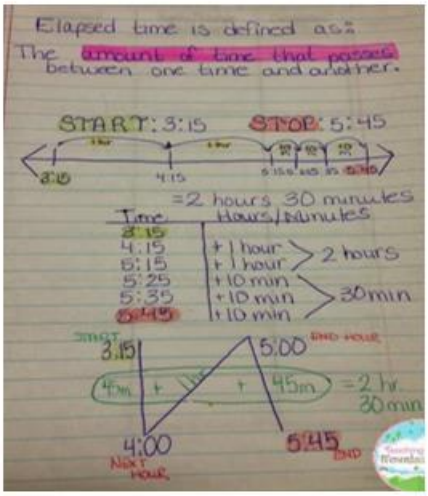
Variations to the Z Method





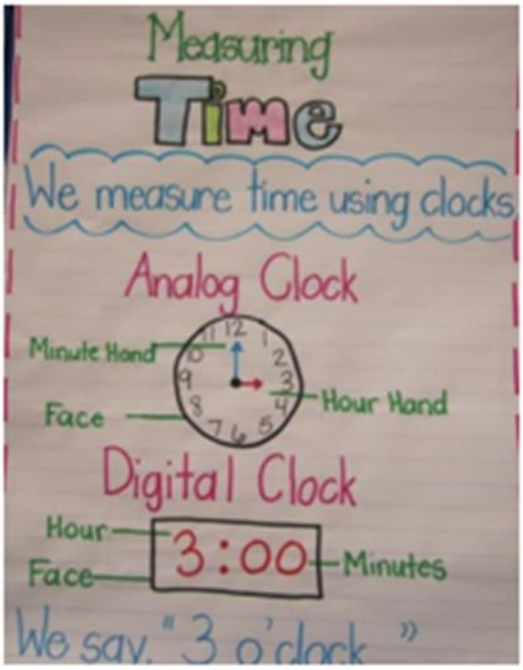


Sample showing the solution for an elapsed time problem using the N Method

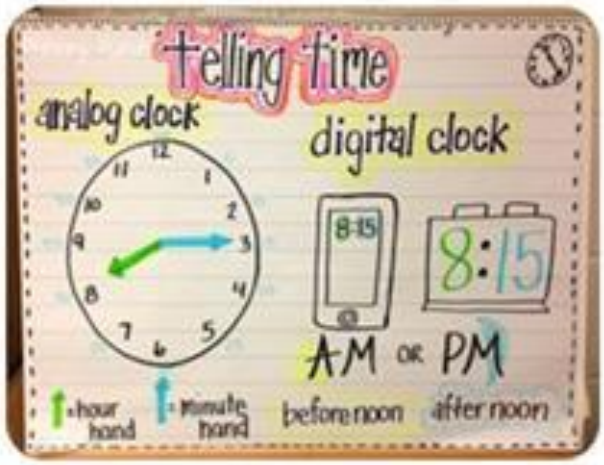


Sample show the solutions for elapsed time problem using 3 methods

Useful charts when teaching Time



Telling Time Song - What Time Is It?



Telling Time Anchor Chart - The Curriculum Corner

Anchor Charts - The Teacher Treasury

Internet References

http://www.teachingwithamountainview.com/2012/10/elapsed-time-troubles.html#_a5y_p=3557264

http://www.elementaryamc.com/2014/02/tried-it-tuesday-elapsed-time-inquiry.html

http://www.teachjunkie.com/math-subject/math-time-to-the-hour/

https://www.youtube.com/watch?v=LafAMJ4TTJw

https://www.youtube.com/watch?v=jG-Yj-m-JjM

https://www.youtube.com/watch?v=ejaOtTZbyEM

https://www.pinterest.com/pin/279152876878822224/

Resources

- Time Number line
- Blank T-charts
- Blank Z-Method charts

Further Reading

Constance Kamii, and Kelly A. Russell. "Elapsed Time: Why Is It So Difficult to Teach?" Journal for Research in Mathematics Education 43.3 (2012): 296-315. Web.